Link Performance Bounds of MIMO Transmit Beamforming Systems Operating over Generalized Fading Channels

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***Abstract*—In this letter, we prove that link performance bounds of multiple-input multiple-output transmit beamforming systems employing maximal ratio combining at the receiver and operating over arbitrary ergodic fading channels can be obtained by analyzing receiver diversity systems operating over specially constructed virtual radio channels.**

***Index Terms*—Generalized fading distributions, maximal ratio combining, MIMO systems, transmit beamforming.**

I. Introduction

Multiple-input multiple-output (MIMO) transmission systems employing multiple antennas at both sides of the transceiver are able to provide both significant diversity and multiplexing gains under proper signal-processing strategies. If channel state information (in the form of a MIMO channel matrix $\mathbf{H}$) is known at the transmitter, the available power can be split among the transmitting antennas appropriately in order to maximize the signal-to-noise ratio (SNR) at the receiver. This method is known as transmit beamforming (TB). If maximal ratio combining (MRC) is applied at the receiver then the effective SNR in the TB MIMO system is proportional to the maximal eigenvalue of the Gramian matrix $\mathbf{W} = \mathbf{H}\mathbf{H}^{\mathrm{H}}$ [1], which reduces to the Wichart matrix [2] if the elements of $\mathbf{H}$ are proper Gaussian. The exact eigenvalue distribution of $\mathbf{H}\mathbf{H}^{\mathrm{H}}$ has been reported only for the Rayleigh and Rician fading models (see, for example, [1], [3]- [4]), and techniques for link performance evaluation have been presented just for these fading scenarios, for example, [3]- [4]. In practice, fading models different from Rayleigh and Rice distributions show often better fits to experimental data [5]- [6].

In this letter, we overcome the uncertainty caused by the unknown eigenvalue distribution of $\mathbf{H}\mathbf{H}^{\mathrm{H}}$ and show that bounds on the statistical distribution of the maximal eigenvalue can be obtained by utilizing only the multivariate fading distributions of the diagonal elements of $\mathbf{W}$. Based on this fact we derive bounds on the outage probability, ergodic capacity, and average error rates for the TB MIMO MRC systems operating over arbitrary ergodic radio channels.

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II. Bounds on the Outage Probability, Average Error Rate, and Ergodic Capacity

A. Preliminaries

We consider the TB MIMO MRC system with N_{T} transmitting and N_{R} receiving antennas operating over the arbitrary ergodic radio channel characterized by the $N_{\mathrm{R}} \times N_{\mathrm{T}}$ matrix $\mathbf{H}$. It was proven in [1] that the maximal SNR γ_{eff} achievable in the system, is proportional to the maximal eigenvalue λ_1 of the matrix $\mathbf{W} = \mathbf{H}\mathbf{H}^{\mathrm{H}}$, that is

$$\gamma_{\mathrm{eff}} = \bar{\gamma}\lambda_1 \tag{1}$$

where $\bar{\gamma}$ is the transmitted SNR. The transmit precoding vector providing γ_{eff} is the unit eigenvector associated with λ_1 [1].

The m th diagonal element of $\mathbf{W}$, w_m, is expressed as

$$w_m = \sum_{i=1}^{N_{\mathrm{T}}} |H_{m,i}|^2, \qquad m = 1, \cdots, N_{\mathrm{R}} \tag{2}$$

where $H_{m,i}$ denotes an element of the matrix $\mathbf{H}$.

B. Link Performance Bounds

Let $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_K$ and $w_1 \geq w_2 \geq \ldots \geq w_K$ ($K = N_{\mathrm{R}}$) be the respective ordered eigenvalues and diagonal elements of $\mathbf{W}$, and $F_{\gamma_{\mathrm{eff}}}(x) = P_{\mathrm{out}}(x) = \Pr\{\gamma_{\mathrm{eff}} \leq x\}$, $P_{\mathrm{aver}} = \lim_{T\to\infty} \frac{1}{T}\int_0^T P_{\mathrm{err}}(\gamma_{\mathrm{eff}}(t))\, dt$, and $C_{\mathrm{erg}} = \lim_{T\to\infty} \frac{B}{T}\int_0^T \log_2(1+\gamma_{\mathrm{eff}}(t))\, dt$ be the respective cumulative distribution function (CDF) of the effective SNR, outage probability (OP), average error rate, and ergodic capacity of the analyzed MIMO channel with the bandwidth B. Then the following proposition is valid.

Proposition: If the TB MIMO MRC system operates over the arbitrary ergodic radio channel represented by the matrix $\mathbf{H}$, the effective SNR, outage probability, average error rate, and ergodic capacity are always bounded by the corresponding metrics of receiver diversity systems employing the selection combining (SC) and MRC methods, and operating over a virtual radio channel with the power gains w_m, $m = 1, \cdots, N_{\mathrm{R}}$, defined by (2), that is

$$\begin{gathered}\gamma_{\mathrm{SC}} \leq \gamma_{\mathrm{eff}} \leq \gamma_{\mathrm{MRC}};\\ F_{\mathrm{MRC}}(x) \leq P_{\mathrm{out}}(x) = F_{\gamma_{\mathrm{eff}}}(x) \leq F_{\mathrm{SC}}(x);\\ P_{\mathrm{aver}_{\mathrm{MRC}}} \leq P_{\mathrm{aver}} \leq P_{\mathrm{aver}_{\mathrm{SC}}};\\ C_{\mathrm{SC}} \leq C_{\mathrm{erg}} \leq C_{\mathrm{MRC}}.\end{gathered} \tag{3}$$

where $\gamma_{\mathrm{SC(MRC)}}, F_{\mathrm{SC(MRC)}}(x) = P_{\mathrm{out}_{\mathrm{SC(MRC)}}}(x)$, $P_{\mathrm{aver}_{\mathrm{SC(MRC)}}}$, and $C_{\mathrm{SC(MRC)}}$ are the respective effective

SNRs, CDFs of the effective SNRs, OPs, average error rates, and capacities of the above virtual radio channel under SC and MRC at the receiver.

Proof: The Horn theorem [7] states that the vector $\lambda = [\lambda_1, \lambda_2, \ldots, \lambda_K]^{\mathrm{T}}$ majorizes the vector $\mathbf{w} = [w_1, w_2, \ldots, w_K]^{\mathrm{T}}$, $\lambda \succ \mathbf{w}$, that is

$$\sum_{m=1}^{k} \lambda_m \geq \sum_{m=1}^{k} w_m, \qquad 1 \leq k \leq K-1,$$
$$\sum_{m=1}^{K} \lambda_m = \sum_{m=1}^{K} w_m. \tag{4}$$

All eigenvalues of $\mathbf{W}$ are positive since $\mathbf{W}$ is Hermitian and positive definite [8]. Thus we obtain from (4) upper and lower bounds on λ_1:

$$w_1 \leq \lambda_1 \leq \sum_{m=1}^{K} w_m. \tag{5}$$

On the left-hand and right-hand sides of (5) we observe the respective effective SNRs of the SC and MRC receiver diversity systems operating over the virtual radio channel with the channel power gains w_m. Then, due to the monotonicity of the functions involved into the evaluation of the error rates and channel capacity, (3) immediately follows from (5). ■

III. Evaluation of Bounds (3)

The evaluation of (3) requires only a knowledge about the statistical distribution of diagonal elements of the matrix $\mathbf{W}$, and a concrete way of the evaluation depends on the concrete fading scenario. In Table I, a few examples of generalized fading distributions of random variables (RVs) $|H_{m,i}|^2$ are given. Scenarios with independent identically distributed (i.i.d.), independent non-identically distributed (i.n.d.), and correlated channel power gains are presented. For each case, we display statistical distributions of w_m and $\sum_{m=1}^{N_{\mathrm{R}}} w_m$, as well as we give references analyzing MRC and SC techniques over the corresponding virtual radio channels. $E[.]$ in Table I denotes the expectation.

We give below two examples of evaluation of (3).

A. Independent and Identically Distributed Generalized Gamma Branches

Let $|H_{m,i}|^2$ follow the generalized gamma (GG) distribution, that is for each $m = 1, \ldots, N_{\mathrm{R}}$ and $i = 1, \cdots, N_{\mathrm{T}}$, the probability density function (PDF) $f_{|H_{m,i}|^2}(x)$ is expressed as

$$f_{|H_{m,i}|^2}(x) = \frac{\alpha x^{\alpha\mu/2-1}}{2\theta^{\mu}\Gamma(\mu)} \exp\left(-\frac{x^{\alpha/2}}{\theta}\right) \tag{6}$$

where α, μ, and θ are the distribution parameters [6], [19]–[22], and $\Gamma(.)$ is the gamma function. This case is given in Table I. By approximating the sum of GG RVs by a single GG RV [19], we find from [19, eq. (22)–(25)] the parameters α_{low}, μ_{low}, and θ_{low} of the GG distribution representing approximately w_m, $m = 1, \ldots, K$, as well as the parameters α_{up}, μ_{up}, and θ_{up} of the GG distribution modeling approximately $\sum_{m=1}^{N_{\mathrm{R}}} w_m$. Then we immediately obtain from (3), (6), and [11] bounds on the OP:

$$P_{\mathrm{out_{SC}}}(\gamma_0) \approx \left[\frac{\gamma\left(\mu_{\mathrm{low}}, (\gamma_0/\bar{\gamma})^{\alpha_{\mathrm{low}}/2}/\theta_{\mathrm{low}}\right)}{\Gamma(\mu_{\mathrm{low}})}\right]^K,$$
$$P_{\mathrm{out_{MRC}}}(\gamma_0) \approx \frac{\gamma\left(\mu_{\mathrm{up}}, (\gamma_0/\bar{\gamma})^{\alpha_{\mathrm{up}}/2}/\theta_{\mathrm{up}}\right)}{\Gamma(\mu_{\mathrm{up}})} \tag{7}$$

where $\gamma(a,t) = \int_0^t x^{a-1}\exp(-x)dx$ is an incomplete gamma function.

$P_{\mathrm{aver_{MRC}}}$ and C_{MRC} can be evaluated via [20] and [22] respectively. Analytical results on the ergodic GG channel capacity and error rates under SC available nowadays (e.g. [20]–[22]), are valid only for integer values of μ_{low}, and it is rather unlikely that the approximation method [19] generally results in integer values of this fading parameter. Thus C_{SC} and $P_{\mathrm{aver_{SC}}}$ can be evaluated numerically, and special techniques allowing to avoid evaluations of infinite-range integrals can be applied. For example, in many practical scenarios, the bit–error probability (BER) conditioned to the SNR γ is expressed in terms of the Gaussian Q-function, $P_{\mathrm{b}}(\gamma) = a_{\mathrm{M}} Q\left(\sqrt{b_{\mathrm{M}}\gamma}\right)$, where a_{M} and b_{M} are parameters defined by the modulation-detection combination [23]. Thus the average BER, $P_{\mathrm{b_{SC}}}$, is expressed as

$$P_{\mathrm{b_{SC}}} = \int_0^{\infty} P_{\mathrm{b}}(x) f_{\mathrm{SC}}(x) dx$$
$$= \frac{a_{\mathrm{M}}\sqrt{b_{\mathrm{M}}}}{2\sqrt{2\pi}} \int_0^{\infty} e^{-b_{\mathrm{M}}x/2} x^{-1/2} F_{\mathrm{SC}}(x) dx \tag{8}$$

where $f_{\mathrm{SC}}(x)$ is the PDF of the effective SNR under SC. The structure of the second integral in (8) allows to apply the Gauss-Laguerre quadrature (GLQ) rule [24], that is

$$P_{\mathrm{b_{SC}}} \approx \frac{a_{\mathrm{M}}}{2\sqrt{\pi}} \sum_{i=1}^{n} q_i x_i^{-1/2} F_{\mathrm{SC}}\left(\frac{2x_i}{b_{\mathrm{M}}}\right) \tag{9}$$

where x_i is the i-th root of the Laguerre polynomial $L_n(x)$, and weights $q_i = \frac{x_i}{(n+1)^2[L_{n+1}(x_i)]^2}$ [24].

Analytical results can be also reported for C_{SC} under low SNR. At low SNR, $\ln(1+x) \approx x$, and

$$C_{\mathrm{SC_{lowSNR}}} \approx B \cdot \frac{K}{\ln 2 (\bar{\gamma}^{\alpha_{\mathrm{low}}/2}\theta_{\mathrm{low}})^{\mu_{\mathrm{low}}} \left[\Gamma(\mu_{\mathrm{low}})\right]^K}$$
$$\times \int_0^{\infty} t^{\mu_{\mathrm{low}} + \frac{2}{\alpha_{\mathrm{low}}} - 1} \exp\left(-\frac{t}{\bar{\gamma}^{\alpha_{\mathrm{low}}/2}\theta_{\mathrm{low}}}\right)$$
$$\times \left[\gamma\left(\mu_{\mathrm{low}}, \frac{t}{\bar{\gamma}^{\alpha_{\mathrm{low}}/2}\theta_{\mathrm{low}}}\right)\right]^{K-1} dt$$
$$= B \cdot \frac{K\bar{\gamma}\theta_{\mathrm{low}}{}^{2/\alpha_{\mathrm{low}}} \Gamma\left(\mu_{\mathrm{low}}K + \frac{2}{\alpha_{\mathrm{low}}}\right)}{\ln 2 \left[\Gamma(\mu_{\mathrm{low}}+1)\right]^K \mu_{\mathrm{low}}}$$
$$\times F_A^{(K-1)}\left(\mu_{\mathrm{low}}K + \frac{2}{\alpha_{\mathrm{low}}}, \mu_{\mathrm{low}}, \cdots, \mu_{\mathrm{low}};\right.$$
$$\left.\mu_{\mathrm{low}}+1, \cdots, \mu_{\mathrm{low}}+1; -1, \cdots, -1\right) \tag{10}$$

where $F_A^{(K-1)}(.)$ is a Lauricella hypergeometric function [25, vol. 3, eq. 7.2.4.54], and the representation of $C_{\mathrm{SC_{lowSNR}}}$ in terms of $F_A^{(K-1)}(.)$ is obtained on the basis of [25, vol. 4, eq. 3.35.7.4]. The structure of the integrand in (10) also allows

TABLE I
STATISTICS OF w_m AND $\sum_{m=1}^{N_{\rm R}} w_m$ FOR SOME GENERALIZED FADING SCENARIOS.

$\lvert H_{m,i}\rvert^2$; parameters of distribution	w_m, $m=1,\ldots,N_{\rm R}$; parameters of distribution	$\sum_{m=1}^{N_{\rm R}} w_m$; parameters of distribution	Analysis of MRC, references	Analysis of SC, references
i.i.d. $\eta-\mu$ RVs [5]; $\{\eta;\mu;E[\lvert H_{m,i}\rvert^2]\}$	i.i.d. $\eta-\mu$ RVs [5]; $\{\eta;N_{\rm T}\cdot\mu;N_{\rm T}\cdot E[\lvert H_{m,i}\rvert^2]\}$	$\eta-\mu$ RV [5]; $\{\eta;N_{\rm T}\cdot N_{\rm R}\cdot\mu;N_{\rm T}\cdot N_{\rm R}\cdot E[\lvert H_{m,i}\rvert^2]\}$	[9]; for integer μ [10]	[11]
i.n.d. $\eta-\mu$ RVs [5]; $\{\eta;\mu;E[\lvert H_{m,i}\rvert^2]\}$	Sum of i.n.d. gamma RVs; distribution is given in [12]. Each w_m can be approximated by a single gamma RV [14]	Sum of i.n.d. gamma RVs; distribution is given in [12]. Can be approximated by a single gamma RV [14]	[12], [13] [15]	[11] [15]
correlated $\eta-\mu$ RVs [5]; $\{\eta;\mu;E[\lvert H_{m,i}\rvert^2]\}$ with integer or half–integer μ	Statistically equivalent to sum of i.n.d. gamma RVs with distribution given in [16]	Statistically equivalent to sum of i.n.d. gamma RVs with distribution given in [16]	[16]	[17]– [18]
i.i.d. $\kappa-\mu$ RVs [5]; $\{\kappa;\mu;E[\lvert H_{m,i}\rvert^2]\}$	i.i.d. $\kappa-\mu$ RVs [5]; $\{\kappa;N_{\rm T}\cdot\mu;N_{\rm T}\cdot E[\lvert H_{m,i}\rvert^2]\}$	$\kappa-\mu$ RV [5]; $\{\kappa;N_{\rm T}\cdot N_{\rm R}\cdot\mu;N_{\rm T}\cdot N_{\rm R}\cdot E[\lvert H_{m,i}\rvert^2]\}$	[9]	[11]
i.i.d. generalized gamma RVs [6], [19]– [22]; $\{\alpha;\mu;\theta\}$	Can be approximated by i.i.d. generalized gamma RVs [19]	Can be approximated by a generalized gamma RV [19]	[20], [22]	[11]; for integer μ [20]– [22]

using the GLQ rule [24] for the approximate evaluation of the integral. For a particular case of two received antennas, (10) reduces to

$$C_{\rm SC_{lowSNR}} \approx \frac{2B\bar{\gamma}{\theta_{\rm low}}^{2/\alpha_{\rm low}}\Gamma\left(2\mu_{\rm low}+\frac{2}{\alpha_{\rm low}}\right)}{\ln 2\left[\Gamma(\mu_{\rm low}+1)\right]^2\mu_{\rm low}} \times {}_2F_1\left(2\mu_{\rm low}+\frac{2}{\alpha_{\rm low}},\mu_{\rm low};\mu_{\rm low}+1;-1\right) \quad (11)$$

where ${}_2F_1(.)$ is the Gauss hypergeometric function [25, vol. 3].

B. Correlated Improper Normally Distributed Branches

Let $H_{m,i}$ be identically distributed zero-mean Gaussian RVs with independent real and imaginary components having different respective variances σ_X^2 and σ_Y^2. We consider the case of constant correlation between the received antennas represented by a correlation coefficient ρ [17]. Then it is seen from (2) that w_m are correlated $\eta-\mu$–distributed RVs with $\mu=N_{\rm T}/2$ and $\eta=\sigma_X^2/\sigma_Y^2$ [5]. This is a scenario presented in Table I. Since μ is either a half-integer or integer, the decomposition [16, eq. (8)–(12)] is valid, and $P_{\rm aver_{MRC}}$ is evaluated on the basis of the method given in [16]. The PDF $f_{\rm MRC}(x)$ and CDF $F_{\rm MRC}(x)$ are given in [13].

The analysis of SC is done on the basis of the multivariate distribution of w_m [17, eq. (8)]. The CDF of w_1 is

$$F_{\rm SC}(x)=\Pr\{w_1\le x\}=F_{\mathbf{w}}(\mathbf{x}) \quad (12)$$

where $F_{\mathbf{w}}(\mathbf{x})$ is the joint CDF of the diagonal elements of $\mathbf{W}$, and $\mathbf{x}=\{\underbrace{x,\ldots,x}_{N_{\rm R}}\}$. Consequently, the PDF $f_{\rm SC}(x)=dF_{\rm SC}(x)/dx$. We evaluate $P_{\rm aver_{SC}}$ by applying the GLQ method (9). $C_{\rm SC}$ and $C_{\rm MRC}$ are evaluated via a numerical integration on the basis of known $f_{\rm SC}(x)$ and $f_{\rm MRC}(x)$.

C. Numerical Results

In Figs. 1–3, we present link performance metrics (obtained via Monte Carlo simulations) and bounds for TB MIMO MRC systems evaluated by using (3) (as it is described in sub-Sections III. A-B), and on the basis of Monte Carlo simulations. The numerical results are given for radio channels with i.i.d. GG branches (GG channels) and for radio channels with correlated improper normally distributed branches (CIN channels) described in sub-Section III. B. In Fig. 1, the OP is shown versus the normalized threshold $\gamma/\bar{\gamma}$ for a few antenna configurations. In Fig. 2, estimates of GG and CIN channel capacities are shown for $N_{\rm T}=2$ and $N_{\rm R}=2$. The estimates of $C_{\rm SC}$ have been obtained via Monte Carlo simulations and via numerical evaluations of the corresponding equations for the ergodic capacity. A few estimates evaluated analytically for low SNR (11) are also reported. Finally, in Fig. 3, BER estimates are shown for the coherent detection of binary phase shift keying (BPSK) with $N_{\rm T}=2$ and $N_{\rm R}=2$.

Although bounds shown in Figs. 1–3 for GG channels are approximate (since they were obtained via the approximation [19]), they agree well with our simulation results in all tested cases. Our numerical results also show that the tightness of bounds (3) depends on all factors involved into the evaluation. The type and parameters of fading distribution as well as antenna configurations affect the tightness.

IV. CONCLUSION

The statistical distribution of the maximal eigenvalue λ_1 of the Gramian matrix is of interest in various MIMO applications. The exact distributions of λ_1 are, however, known only for the Rayleigh and Rice models. In this letter, we overcome this uncertainty and present link performance bounds for TB MIMO MRC systems based solely on the multivariate distribution of diagonal elements of the Gramian matrix. Depending on the concrete fading scenario, the presented technique provides either closed-form expressions or those given in the single-integral form. But in any case, the method allows avoiding time-consuming computer simulations.

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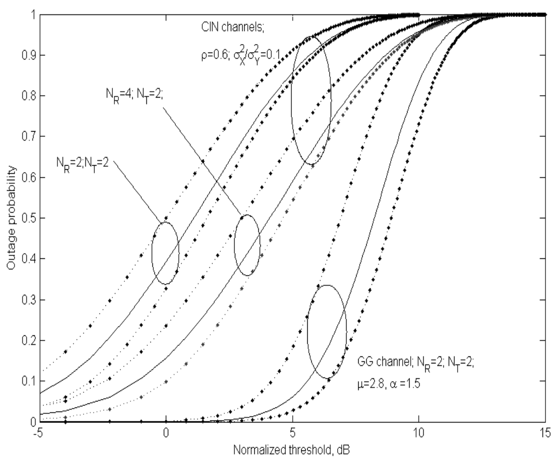

Fig. 1. Outage probability versus normalized threshold $\gamma/\bar{\gamma}$. Solid lines represent numerical estimates of outage probability, dotted lines represent bounds (3), and single points report simulation results.

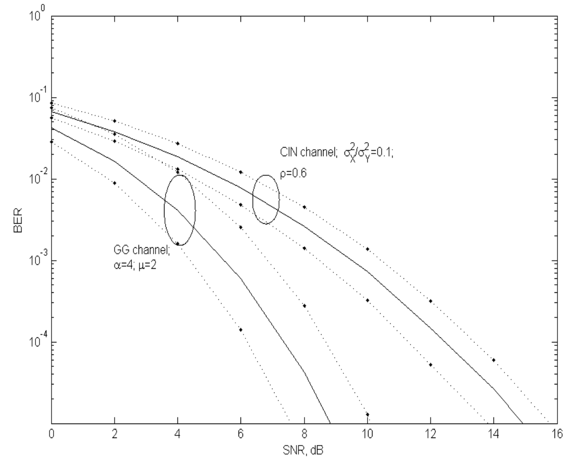

Fig. 3. Bounds on the BER performance, BPSK modulation format, $N_{\rm T} = 2$ and $N_{\rm R} = 2$. Solid lines represent numerical BER estimates, dotted lines represent bounds (3), and single points report simulation results.

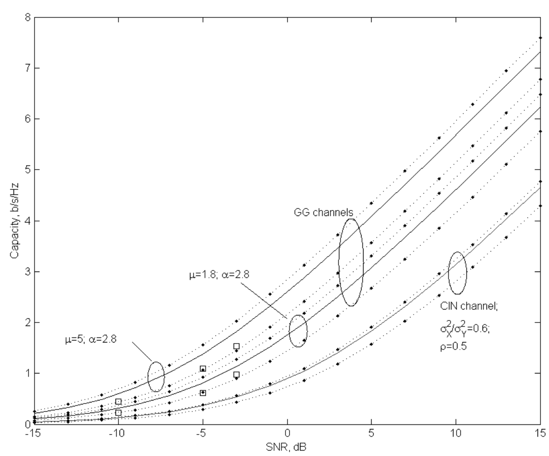

Fig. 2. Capacity per unit bandwidth, $N_{\rm T} = 2$ and $N_{\rm R} = 2$. Solid lines represent numerical estimates of capacity, dotted lines represent bounds (3), and single points report simulation results. Squares represent estimates for low SNR (11).

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