Performance Analysis of Communication Systems over Generalized $\alpha - \lambda - \eta - \mu$ Fading Radio Channels

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***Abstract*—In this paper, we recognize the relation between the generalized $\alpha - \lambda - \eta - \mu$ and $\eta - \mu$ fading distributions. We present an approximate technique providing the reduction of the $\alpha - \lambda - \eta - \mu$ distribution to the generalized gamma distribution. For integer values of the fading parameter μ, we prove that the probability density function (PDF) of the $\alpha - \lambda - \eta - \mu$ distribution is expressed via a linear combination of PDFs of the generalized gamma distributions. The presented results can be used for the evaluation of error rates over $\alpha - \lambda - \eta - \mu$ fading and channel capacity. We give a full statistical characterization of the multivariate $\alpha - \lambda - \eta - \mu$ distribution and present a few examples demonstrating the applicability of the derived results including multi-antenna systems employing different receiver diversity methods.**

I. Introduction

The generalized $\alpha - \lambda - \eta - \mu$ distribution was recently introduced in [1] for modeling multipath (small-scale) fading in a non-line-of-sight propagation environment. This statistical model assembles previously presented $\alpha - \mu$ [2] and $\eta - \mu$ [3] general fading distributions and include them as particular cases. Obviously, the Rayleigh, Nakagami-m, one-sided Gaussian, generalized gamma, and Weibull distributions are also special cases of the $\alpha - \lambda - \eta - \mu$ distribution.

Such widely used small-scale fading models as the Rayleigh or Nakagami-m fading distributions assume a homogeneous propagation environment where in-phase (I) and quadrature (Q) components of the fading signal are independent and have equal powers. The real fading environment is, however, non-homogeneous [4]. More advanced fading distibutions (for example the $\eta - \mu$ [3] and $\alpha - \lambda - \eta - \mu$ models [1]) take into account the non-homogeneous structure of the propagation medium and consider the I and Q components correlated and (or) having different powers. The $\alpha - \lambda - \eta - \mu$ fading model additionally takes into account possible nonlinearities of the propagation medium via the fading parameter α. A large number of examples given in [3] and [1], prove that these fading models show better fits to experimental data than some widely applied models, for example, the Nakagami-m distribution.

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The $\alpha - \lambda - \eta - \mu$ model treats the fading signal as the composition of the multipath clusters in the nonlinear and non-homogeneous propagation medium where the Gaussian I and Q components within each cluster are both correlated and have different powers. The moments, probability density and cumulative distribution functions (PDF and CDF) of one-dimensional distribution were obtained in [1]. In this paper, we evaluate such performance metrics as the bit-error rate, channel capacity, and outage probability. We also derive the multivariate PDF and CDF for arbitrary correlation models and apply these results for the performance evaluation of multi-antenna systems employing maximal-ratio combining (MRC) and selection combining (SC) over $\alpha - \lambda - \eta - \mu$ branches.

II. The $\alpha - \lambda - \eta - \mu$ Fading Distribution

A. One-dimensional Model

The fading envelope R is a nonlinear function of the composition of n multipath clusters with the Gaussain I and Q component within each cluster [1], that is

$$R^\alpha = \sum_{i=1}^{n} \left(X_i^2 + Y_i^2\right) \quad (1)$$

where $\alpha > 0$ is a power parameter capturing nonlinear propagation effects, X_i and Y_i are correlated zero-mean Gaussain variables with the respective powers $E\{X_i^2\} = \sigma_X^2$ and $E\{Y_i^2\} = \sigma_Y^2$. The parameter $\eta = \sigma_X^2/\sigma_Y^2$ characterizes the power ratio of the I and Q components, and the correlation between X_i and Y_j is represented by a correlation coefficient

$$\lambda = \delta_{i,j} \frac{E\{X_i Y_j\}}{\sigma_X \sigma_Y} \quad (2)$$

where $\delta_{i,j}$ is the Kronecker symbol.

The model (1) is directly extended to real values of n due to the reasons given in [3], [1], and a fading parameter μ (that is an real extension of $n/2$) is introduced. The PDF of the $\alpha - \lambda - \eta - \mu$ envelope variable is given in [1, eq. (10)], and the PDF of the power variable γ can be obtained directly after a standard transformation [5]. We give below (see eq.

(5)) an alternative PDF expression resulting from the following Proposition.

Proposition 1: The $\alpha-\lambda-\eta-\mu$ power variable γ is related to an $\eta-\mu$ power variable $\gamma_{\eta-\mu}$ via a nonlinear transform

$$\gamma = \gamma_{\eta-\mu}^{2/\alpha}. \quad (3)$$

Moreover, γ can be approximately represented as (is approximately statistically equivalent to) a generalized gamma variable $g_{\rm gen}$ with the PDF $f_{g_{\rm gen}}(x)$:

$$f_{g_{\rm gen}}(x,m,\vartheta) = \frac{\alpha x^{\alpha m/2-1}}{2\vartheta^m \Gamma(m)} \exp\left(-\frac{x^{\alpha/2}}{\vartheta}\right) \quad (4)$$

where $m = E^2\{R^\alpha\}/{\rm var}\{R^\alpha\} = \left\{\left[(1+1/\mu)\left(\theta_1^2+\theta_2^2\right)+2\theta_1\theta_2\right]/\left(\theta_1+\theta_2\right)^2-1\right\}^{-1}$ with $\theta_{1(2)} = \frac{2}{c_1+c_2-(+)\sqrt{(c_2-c_1)^2+4a^2}}$ where $a = \frac{\lambda}{2\sigma_X\sigma_Y(1-\lambda^2)}$, $c_1 = \left[2\sigma_X{}^2\left(1-\lambda^2\right)\right]^{-1}$, and $c_2 = \left[2\sigma_Y{}^2\left(1-\lambda^2\right)\right]^{-1}$. In (4), $\vartheta = \mu(\theta_1+\theta_2)\bar{\gamma}^{\alpha/2}/m$ where $\bar{\gamma} = E_{\rm b}/N_0$ is the transmitted signal-to-noise ratio (SNR) per bit.

Proof: It is seen from (1) that the $\alpha/2$th power of the power variable, $\gamma^{\alpha/2}$, is the sum of two correlated gamma variables with the same shape parameter μ (the real extension of $n/2$) and respective scale parameters $2\sigma_X^2$ and $2\sigma_Y^2$. But it is proven in [6] that the sum of two correlated gamma variables with the given scale parameters is statistically equivalent to the sum of two independent gamma variables with the defined above scale parameters θ_1 and θ_2. By the definition, this sum can be viewed as an $\eta-\mu$ power variable of format 1 [3]. This fact proves the validity of (3). Using (3) and the standard PDF transformation procedure [5], we obtain an alternative expression for the PDF of the $\alpha-\lambda-\eta-\mu$ power variable:

$$f_\gamma(x) = \frac{\alpha\sqrt{\pi}\cdot\exp\left[-\left(\frac{x}{\bar{\gamma}}\right)^{\alpha/2}\frac{(1/\theta_1+1/\theta_2)}{2}\right]}{2\bar{\gamma}^{\alpha/2(\mu+1/2)}\Gamma(\mu)} \times \frac{x^{\alpha/2(\mu+1/2)-1}{\rm I}_{\mu-1/2}\left[\left(\frac{x}{\bar{\gamma}}\right)^{\alpha/2}\frac{(1/\theta_2-1/\theta_1)}{2}\right]}{(\theta_1\theta_2)^\mu\left(1/\theta_2-1/\theta_1\right)^{\mu-1/2}} \quad (5)$$

where $\Gamma(a) = \int_0^\infty t^{\alpha-1}e^{-t}dt$ is the gamma function.

It is proven in [7] that the sum G of two independent gamma variables (that is the $\eta-\mu$ power variable [3]) can be approximated by a single gamma variable with the shape parameter $p = \frac{E^2\{G\}}{E\{G^2\}-E^2\{G\}}$ and scale parameter $\vartheta = E\{G\}/p$. The values of m and ϑ in (4) are defined on the basis of this rule. ■

Obviously, the CDF $F_\gamma(z)$ of the distribution (3) is approximately that of the generalized gamma distribution (4):

$$F_\gamma(z) \approx \frac{\gamma(m, z^{\alpha/2}/\vartheta)}{\Gamma(m)} \quad (6)$$

where $\gamma(a,t) = \int_0^t x^{a-1}\exp(-x)dx$ is an incomplete gamma function.

Corollary 1: The moment generating function of the $\alpha-\lambda-\eta-\mu$ distribution $M_\gamma(s) = E\{\exp(-s\gamma)\}$ is approximately

$$M_\gamma(s) \approx \frac{1}{\Gamma(m)} \times H_{22}^{11}\left[s\vartheta^{2/\alpha}\,\middle|\,\begin{matrix}(1-m,2/\alpha), & (1,0)\\ (0,1) & (0,0)\end{matrix}\right] \quad (7)$$

where $H\,[.]$ is the Fox H-function [8] that can be expressed in terms of the Meijer G-function if α is a rational number (i.e. $\alpha = 2k/l$) [8]:

$$H_{22}^{11}\left[s\vartheta^{2/\alpha}\,\middle|\,\begin{matrix}(1-m,2/\alpha), & (1,0)\\ (0,1) & (0,0)\end{matrix}\right] = \sqrt{\frac{k}{l}}(2\pi)^{1-(k+l)/2}l^m \times G_{lk}^{kl}\left[\frac{\left(s\vartheta^{2/\alpha}\right)^k l^l}{k^k}\,\middle|\,\begin{matrix}\Delta(l,1-m)\\ \Delta(k,0)\end{matrix}\right] \quad (8)$$

where $\Delta(k,a) = \frac{a}{k}, \frac{a+1}{k}, \ldots, \frac{a+k-1}{k}$.

Proof: The expressions (7)-(8) are derived on the basis of the technique presented in [9], and they are equivalent to the MGF expression given in this work although the form of the representation is another because we use the other form of representation of the Meijer G-function given in [8]. ■

Proposition 2: For integer values of μ, the PDF of the $\alpha-\lambda-\eta-\mu$ distribution (5) is expressed in terms of elementary functions as a finite sum of the scaled generalized gamma PDFs:

$$f_\gamma(x) = \frac{(-1)^\mu\alpha}{2\bar{\gamma}^{\alpha/2}\Gamma(\mu)\left(\theta_1\theta_2\right)^\mu\theta_{\rm d}^{2\mu-1}}\left\{\sum_{i=0}^{\mu-1}(\mu-i)_{\mu-1} \times f_{g_{\rm gen}}(x,m_i,\vartheta_1)\theta_{\rm d}^i - \sum_{i=0}^{\mu-1}(\mu-i)_{\mu-1}(-1)^i \times f_{g_{\rm gen}}(x,m_i,\vartheta_2)\theta_{\rm d}^i\right\} \quad (9)$$

where $(a)_k$ i the Pochhammer symbol [8], $\theta_{\rm d} = 1/\theta_2 - 1/\theta_1$, $m_i = i+1$, $\vartheta_1 = \bar{\gamma}^{\alpha/2}\theta_2$, $\vartheta_2 = \bar{\gamma}^{\alpha/2}\theta_1$, and $f_{g_{\rm gen}}(.)$ is the generalized gamma PDF (4).

Proof: Eq. (9) is obtained on the basis of the finite series expansion of $I_{\mu-1/2}$ in (5) with integer values of μ [12, eq. (8.467)]. ■

With the help of (9) one can apply many results on generalized gamma distribution (e.g. [9] – [11]) to the performance evaluation over generalized $\alpha-\lambda-\eta-\mu$ fading with integer values of μ.

Corollary 2: For integer values of μ and rational $\alpha = 2k/l$, an exact MGF expression in terms of the finite sum of the Meijer G-function is available:

$$M_\gamma(s) = (-1)^\mu\sqrt{\frac{k}{l}}\frac{(2\pi)^{1-(k+l)/2}\alpha l^m}{2\bar{\gamma}^{\alpha/2}\Gamma(\mu)\left(\theta_1\theta_2\right)^\mu\theta_{\rm d}^{2\mu-1}}\left\{\sum_{i=0}^{\mu-1}(\mu-i)_{\mu-1} \times \theta_{\rm d}^i G_{lk}^{kl}\left[\frac{\left(s\vartheta_1^{2/\alpha}\right)^k l^l}{k^k}\,\middle|\,\begin{matrix}\Delta(l,1-m_i)\\ \Delta(k,0)\end{matrix}\right]\right.$$

$$
-\sum_{i=0}^{\mu-1}(-1)^i(\mu-i)_{\mu-1}\theta_{\rm d}^i
$$

$$
\times G_{lk}^{kl}\left[\frac{\left(s\vartheta_2^{2/\alpha}\right)^k l^l}{k^k}\left|\begin{array}{c}\Delta(l,1-m_i)\\ \Delta(k,0)\end{array}\right.\right]\Bigg\}. \quad (10)
$$

Corollary 3: For integer values of μ, the CDF of the $\alpha-\lambda-\eta-\mu$ distribution is expressed in terms of elementary functions:

$$
F_\gamma(z)=\frac{(-1)^\mu\alpha}{2\bar{\gamma}^{\alpha/2}\Gamma(\mu)\left(\theta_1\theta_2\right)^\mu\theta_{\rm d}^{2\mu-1}}\left\{\sum_{i=0}^{\mu-1}(\mu-i)_{\mu-1}\theta_{\rm d}^i\right.
$$

$$
\times\left(1-\exp(-z^{\alpha/2}/\vartheta_1)\sum_{l=0}^{i}\frac{z^{\alpha l/2}}{\vartheta_1^l l!}\right)
$$

$$
-\sum_{i=0}^{\mu-1}(-1)^i(\mu-i)_{\mu-1}\theta_{\rm d}^i
$$

$$
\left.\times\left(1-\exp(-z^{\alpha/2}/\vartheta_2)\sum_{l=0}^{i}\frac{z^{\alpha l/2}}{\vartheta_2^l l!}\right)\right\}. \quad (11)
$$

B. PDF and CDF of the Multivariate Distribution

The PDF of the multivariate $\alpha-\lambda-\eta-\mu$ distribution $f_\gamma(z_1,z_2,\ldots,z_n)$ can be defined on the basis of (3) and an PDF expression of the multivariate $\eta-\mu$ distribution $f_{\gamma_{\eta-\mu}}(z_1,z_2,\ldots,z_n)$ [6], [13] by using the standard transformation algorithm [5]:

$$
f_\gamma(z_1,z_2,\ldots,z_n)=\left(\frac{\alpha}{2}\right)^n\prod_{i=1}^{n}z_i^{\alpha/2-1}
$$

$$
\times f_{\gamma_{\eta-\mu}}(z_1^{\alpha/2},z_2^{\alpha/2},\ldots,z_n^{\alpha/2}). \quad (12)
$$

Depending on a given correlation model one must insert in (12) a proper expression for the multivariate $\eta-\mu$ PDF $f_{\gamma_{\eta-\mu}}(\mathbf{z})$. A most general $f_{\gamma_{\eta-\mu}}(\mathbf{z})$ expression for an arbitrary correlation model is given in [6, eq. (8)]. Simpler expressions for special cases have been also reported. Such are particular cases of the bivariate distribution [15, eq. (13)], constant [13, eq. (7)], and exponential correlation models [6, eq. (10)]. All these multivariate $\eta-\mu$ distributions are given in terms of a normalized correlation matrix $\mathbf{R}$ with the elements $R_{i,j}=\frac{E\{\gamma_i^{\alpha/2}\gamma_j^{\alpha/2}\}-E\{\gamma_i^{\alpha/2}\}E\{\gamma_j^{\alpha/2}\}}{\sqrt{\mathrm{var}\{\gamma_i^{\alpha/2}\}\mathrm{var}\{\gamma_j^{\alpha/2}\}}}$. If the correlation properties of the vector $\alpha-\lambda-\eta-\mu$ variable are given in terms of the normalized correlation matrix $\tilde{\mathbf{R}}$ with the elements $\tilde{R}_{i,j}=\frac{E\{\gamma_i\gamma_j\}-E\{\gamma_i\}E\{\gamma_j\}}{\sqrt{\mathrm{var}\{\gamma_i\}\mathrm{var}\{\gamma_j\}}}$, a relation between $R_{i,j}$ and $\tilde{R}_{i,j}$ must be identified. This can be done numerically on the basis of the previously derived results on the $\eta-\mu$ distribution [3, eq. (21)] and [15, eq. (13)]. We note that the joint moments $E\{\gamma_i\gamma_j\}$ can be directly obtained from the bivariate $\eta-\mu$ PDF [15, eq. (13)] for arbitrary values of the correlation coefficient between two $\eta-\mu$ variables while (17) in [15] is valid only for restricted values of this parameter. It is also worth noting that the formulas in [3] and [15] are expressed via parameters H and h of the $\eta-\mu$ distribution, which for the case of the format 1 are expressed in terms of the fading parameter $\tilde{\eta}$ characterizing the power ratio of the independent I and Q components of the signal: $H=(\tilde{\eta}^{-1}-\tilde{\eta})/4$ and $h=(2+\tilde{\eta}^{-1}+\tilde{\eta})/4$ [3]. It follows from Proposition 1 that $\tilde{\eta}=\theta_2/\theta_1$. Generally, $\tilde{\eta}\neq\eta$ since η expresses the power ratio of the dependent I and Q components.

An example of the dependence $R_{i,j}=f(\tilde{R}_{i,j})$ is shown in Fig. 1 for $\mu=2$ and a few values of the fading parameter α.

The CDF $F_\gamma(z_1,\ldots,z_n)=Pr(\gamma_1\le z_1,\ldots,\gamma_n\le z_n)=F_{\gamma_{\eta-\mu}}(z_1^{\alpha/2},\ldots,z_n^{\alpha/2})$.

III. Performance Analysis of Communication Systems

A. Single-branch Transmission

For a single branch transmission, an expression for the average bit-error rate over the generalized gamma fading channel was obtained in [10, eq. (14)] under the condition that the bit-error rate for the additive white Gaussian noise channel is expressed as

$$
P_{\rm b}=\frac{\Gamma(b,a\gamma_{\rm eff.})}{2\Gamma(b)} \quad (13)
$$

where $\gamma_{\rm eff.}$ is the effective SNR, and $\Gamma(b,t)=\int_t^\infty z^{b-1}\exp(-z)dz$ is the complementary incomplete gamma-function. Formula (13) is a generic bit-error rate expression for binary modulation schemes [14, eq. (8.100)]. It involves such modulation formats as binary phase-shift keying (BPSK) ($a=1$, $b=0.5$), differential BPSK (DBPSK) ($a=1$, $b=1$), binary frequency-shift keying (BFSK) ($a=0.5$, $b=0.5$), and non-coherent BFSK ($a=0.5$, $b=1$). Eq. (13) involves as a particular case the Gaussian Q-function $Q(b_{\rm M}\sqrt{x})=\Gamma(1/2,b_{\rm M}^2x/2)/\left[2\Gamma(1/2)\right]$, and thus scaled versions of (13) may be used for the BER evaluation for a large variety of M-ary modulation formats where $P_{\rm b}=a_{\rm M}Q(b_{\rm M}\sqrt{x})$ with $a_{\rm M}$ and $b_{\rm M}$ defined by the modulation-detection combination [14].

On the basis of Proposition 1 one can also obtain an approximate expression for the ergodic channel capacity [16], [11]:

$$
C_{\rm erg}=B\cdot E\{\log_2(1+\gamma)\}
$$

$$
\approx B\int_0^\infty\log_2(1+\gamma)f_{g_{\rm gen}}(\gamma)d\gamma=\frac{\alpha}{2\Gamma(m)\vartheta^m}
$$

$$
\times\frac{B}{k\ln 2\cdot}\frac{l^{1/2}}{(2\pi)^{k+l/2-3/2}}G_{(2k)(2k+l)}^{(2k+l)(k)}
$$

$$
\left[(l\cdot\vartheta)^{-l}\left|\begin{array}{ccc}\Delta(k,-\chi), & \Delta(k,1-\chi) & \\ \Delta(l,0), & \Delta(k,-\chi), & \Delta(k,-\chi)\end{array}\right.\right] \quad (14)
$$

where B is the channel bandwidth, and $\chi=m\cdot k/l$. The integral in (14) is solved via the representation of $\exp(.)$ and $\ln(1+\gamma)$ in terms of the Meijer G-function [8, eq. 8.4.3.1] and [8, eq. 8.4.6.5], respectively, and application of [8, eq. 2.24.1.1].

For integer values of μ, the PDF $f_\gamma(x)$ is expressed as a linear combination of PDFs of generalized gamma distributions (see (9)). Thus in this case, an exact expression for the ergodic channel capacity can be derived on the basis of (9) and (14).

B. Multi-antenna Systems

1) Maximal Ratio Combining over Independent Branches: We consider a diversity scheme applying the MRC technique [14] over L independent branches. We use the MGF-based approach [14] and obtain a finite-integral expression for P_{aver}:

$$P_{\text{aver}} = \frac{a_{\text{M}}}{\pi} \int_0^{\pi/2} \prod_{i=1}^{L} M_{\gamma_i} \left(\frac{b_{\text{M}}^2}{2\sin^2\theta} \right) d\theta \tag{15}$$

where $M_{\gamma_i}(s)$ is the MGF of the SNR over the i *th* branch expressed by (7) for arbitrary fading parameters or by (10) for integer values of the fading parameters μ_i at the individual branches.

2) Selection Combining over Correlated Branches: We consider the evaluation of the outage probability $P_{\text{out}}(q)$ of a communication system employing the SC method and operating over the correlated branches. The outage probability under SC is expressed as

$$\begin{aligned} P_{\text{out}}(q) &= \Pr\left(\max_{1 \le i \le L} \text{SNR}_{\text{r}_i} \le q \right) = F_{\gamma}(\mathbf{q}) \\ &= F_{\gamma_{\eta-\mu}} \underbrace{\left(q^{\alpha/2}, \ldots, q^{\alpha/2}\right)}_{L} \end{aligned} \tag{16}$$

where SNR_{r_i} is the received SNR over the i th branch (i.e. the product of the transmitted SNR and channel power gain), and $\mathbf{q} = \{\underbrace{q, \ldots, q}_{L}\}$.

IV. Numerical Results

In this Section, we present a few performance metrics evaluated on the basis of the results obtained in this paper. Both analytical estimates and numerical results are given. Numerical estimates were obtained via Monte-Carlo simulations where the $\alpha - \lambda - \eta - \mu$ variable was generated as the $(2/\alpha)$th power of the sum of two correlated gamma variables (see (1)). We also applied a simpler way based on Proposition 1 where the sum of independent gamma variables with properly chosen parameters was used. Both methods provide identical results. Additionally, we compared the analytical estimates with the results obtained via the numerical integration of the corresponding equations. Under all scenarios considered in this work, we observed a very good agreement between the estimates obtained in different ways.

In Fig. 2, we show the ergodic channel capacity (per unit bandwidth) for the case of the single-antenna transmission. The curves are given for a few values of the fading parameters. For $\mu = 1$, we use an exact analytical method based on the application of (9) and (14). For $\mu = 2.8$, an approximate solution is obtained by the application of (4) and (14). The presented results allow estimating impacts of the fading parameters on the channel capacity. In Fig. 3, estimates of the average BER are given for the cases of the single-antenna transmission and for MRC over two i.i.d. branches. The curves are given for two modulation formats, BPSK and DBPSK, and for $\mu = 1$ and $\mu = 2.8$. In this case, $\eta = 0.1$ and $\lambda = 0.5$. Finally, the curves in Fig. 4 present the outage probability versus the normalized outage threshold $(\gamma/\bar{\gamma})$ for the multi-antenna system employing SC over independent and correlated branches. We consider three- and four-branch receivers. The fading parameters are: $\alpha = 4$, $\lambda = 0.5$, $\eta = 0.1$, and $\mu = 1$. We consider the case of the constant correlation between branches with the correlation coefficient $\rho = R_{i,j} = 0.6$ as well as the case of independent branches.

In all cases considered we observe a good accuracy of the approximation (4).

V. Conclusion

In this paper, we present results that can be useful for the performance evaluation of communication systems operating over generalized $\alpha - \lambda - \eta - \mu$ fading. We recognize the relation between the $\alpha - \lambda - \eta - \mu$ variable and the $\eta - \mu$ variable: the $\alpha - \lambda - \eta - \mu$ fading envelope is a nonlinear function ($1/\alpha$th power) of the $\eta - \mu$ power variable. Thus the $\alpha - \lambda - \eta - \mu$ statistical model can be referred to as a generalized $\eta - \mu$ distribution. The recognized relation allowed us deriving closed-form expressions for the multivariate PDF and CDF that can be employed in the performance evaluation of multi-antenna systems.

For integer values of the fading parameter μ, we present PDF and CDF expressions in terms of elementary functions, and an MGF expression in terms of the Meijer G-function. For arbitrary values of the fading parameters, we propose an approximate method reducing the $\alpha - \lambda - \eta - \mu$ distribution to the generalized gamma distribution. All the derived results give possibilities of analyzing performances of communication systems operating over $\alpha - \lambda - \eta - \mu$ fading without cumbersome and time-consuming computer simulations.

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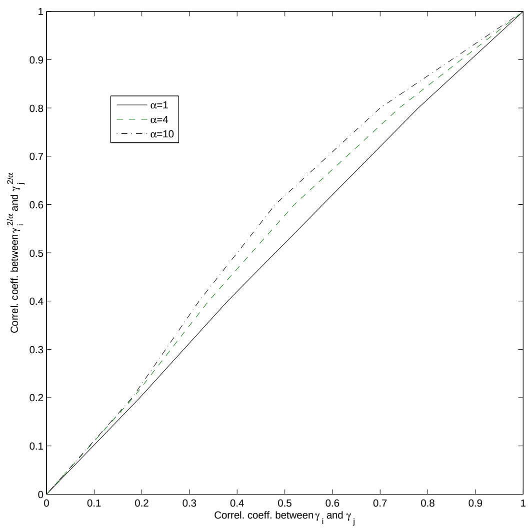

Fig. 1. Relations between $R_{i,j}$ and $\tilde{R}_{i,j}$ for $\mu = 2$ and a few values of fading parameter α.

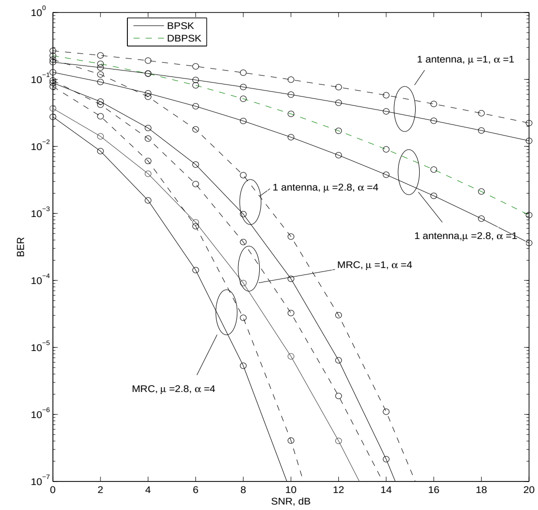

Fig. 3. Average BER for single-antenna transmission and for MRC over two independent branches. Single points report simulation results.

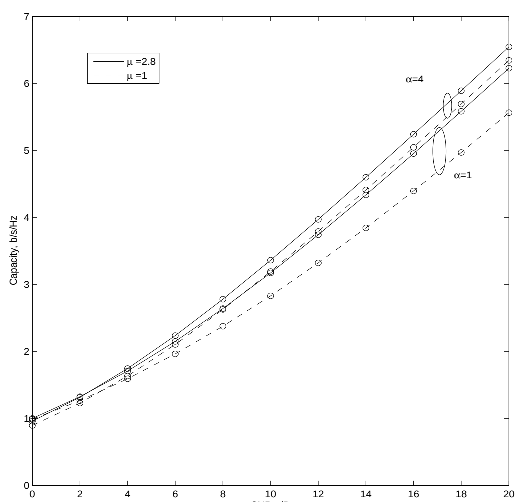

Fig. 2. Ergodic channel capacity per unit bandwidth (b/s/Hz), single-antenna transmission. Single points report simulation results.

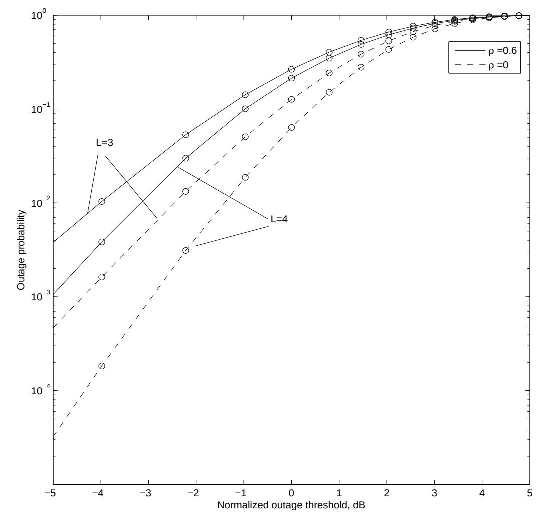

Fig. 4. Outage probability versus normalized outage threshold. Single points report simulation results.

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