Outage Probability Analysis in Generalized Fading Channels with Co-Channel Interference and Background Noise: $\eta-\mu/\eta-\mu$, $\eta-\mu/\kappa-\mu$, and $\kappa-\mu/\eta-\mu$ Scenarios

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***Abstract*—In this paper, we present expressions for the cumulative distribution function (CDF) of a specially constructed random variable (RV) represented by the ratio of two generalized RVs. The obtained theoretical results are used to evaluate the outage probability in scenarios with η-μ–faded signals of interest (SoI), η-μ– or κ-μ–faded co-channel interference (CCI), and background white Gaussian noise. Our results are applicable also to scenarios where the SoI passes through the κ-μ fading channel, and the interfering signals are η-μ–faded. The derived results can be used if all parameters μ_i of the η-μ models representing the statistical distributions of either the SoI components or CCI components are integers. We prove, in particular, that in the former case, the CDF is expressed in terms of elementary functions.**

***Index Terms*—Co-channel interference, η-μ and κ-μ fading distributions, outage probability, ratio of random variables.**

I. Introduction

Analysis of communication systems with co-channel interference (CCI) is of interest in many practical applications, including cellular mobile networks, ad hoc networks, and cognitive radio systems. This fact caused a large amount of research activity in this area, and nowadays analytical results characterizing interference in systems with CCI have been reported for many combinations of fading models that experience the signal of interest (SoI) and CCI. Some studies analyze interference-limited scenarios (that is, scenarios without background noise), while others take background noise into account. Detailed analysis of interference–limited scenarios for different SoI and CCI fading models can be found in books [1, Chapter 3]–[2, Chapter 10]. Scenarios with background noise taken into account were considered, for example, in several studies [3]–[5]. More precisely, Nakagami-m–faded components of the SoI and equal-power Nakagami-m–faded components of CCI (Nakagami-m/Nakagami-m scenarios) were assumed in [3], whereas Nakagami-m/Rayleigh and Rice/ Rayleigh scenarios were analyzed in [4], and Rice/Rice cases were investigated in [5].

The η-μ and κ-μ fading distributions were recently introduced by M. D. Yacoub [6] for modeling non-line-of-sight and line-of-sight propagation effects, respectively. Both statistical models are generalized fading distributions that include many well-known fading models (for example, the Nakagami-m, Nakagami-q, and Nakagami-n distributions). Moreover, these models fit better to measurement data than do the commonly used fading models [6].

The outage probability (OP) is an important metric characterizing the signal-to-interference ratio in systems with CCI. Recently, several studies analyzed the OP in η-μ and κ-μ fading channels with CCI. An approximate integral-form expression was derived in [7] for equal-gain combining at the receiver. An elegant solution in terms of elementary functions was found in [8], where the author considered interference-limited scenarios over η-μ and κ-μ fading channels while assuming a number of η-μ–faded interfering signals with integer values of the fading parameter μ_i. Expressions for the OP in scenarios assuming the η-μ–faded SoI, Rayleigh–faded CCI, and background noise, were recently derived in [9].

In this paper, we obtain analytical results that can be applied to an analysis of the OP in wireless communication systems with CCI and background noise. For example, on the basis of the results presented here, the OP can be evaluated analytically in η-μ/η-μ, η-μ $/\kappa$-μ, and κ-μ/η-μ fading scenarios with integer parameters μ_i of the η-μ distributions representing the SoI or CCI fading models. If all parameters μ_i of the η-μ distributions characterizing the SoI components are integers, the OP is expressed in terms of elementary functions.

II. Cumulative Distribution Functions of the Ratio of Generalized Random Variables

A. System Model

In this study, we analyze the statistical distribution of a random variable (RV) γ, which is expressed as

$$\gamma = \frac{\sum_{i=1}^{N_\mathrm{S}} s_i}{\sum_{i=1}^{N_\mathrm{I}} c_i + \sigma^2} \quad (1)$$

where s_i and c_i are η-μ (κ-μ)–distributed power variables, and $\sigma^2 > 0$ is a positive constant. Under some scenarios, γ may be viewed as the signal-to-interference-plus-noise ratio (SINR) in communication systems with CCI and background white Gaussian noise with the variance σ^2. In this case, N_S and N_I in (1) characterize the number of the SoI and CCI components, respectively.

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The probability density function (PDF) of the η-μ power variable $\gamma_{\eta-\mu}$, $f_{\gamma_{\eta-\mu}}$, is expressed as [6]

$$f_{\gamma_{\eta-\mu}}(x) = \frac{2\sqrt{\pi}\mu^{\mu+\frac{1}{2}}h^{\mu}x^{\mu-\frac{1}{2}}}{\Gamma(\mu)H^{\mu-\frac{1}{2}}\Omega_\eta^{\mu+\frac{1}{2}}}\exp\left(-\frac{2\mu x h}{\Omega_\eta}\right) \times I_{\mu-\frac{1}{2}}\left(\frac{2\mu H x}{\Omega_\eta}\right) \quad (2)$$

where $\Omega_\eta = E\{\gamma_{\eta-\mu}\}$, $\mu = \frac{\Omega_\eta^2}{2\mathrm{var}\{\gamma_{\eta-\mu}\}}\left[1+\left(\frac{H}{h}\right)^2\right]$ (with $E\{.\}$ and $\mathrm{var}\{.\}$ denoting the expectation and variance respectively), $\Gamma(.)$ is the gamma function, and $I_\alpha(.)$ is the modified Bessel function of the first kind of the order α. The statistical model (2) comprises two different fading scenarios (formats) with different physical meanings of the parameter η and different definitions of the parameters H and h. In format (1), the parameter $0 < \eta < \infty$ represents the power ratio of the in-phase and quadrature components of the fading signal in each multipath cluster, and the parameters H and h are defined as

$$H = (\eta^{-1} - \eta)/4 \qquad \text{and} \qquad h = (2 + \eta^{-1} + \eta)/4. \quad (3)$$

In format 2, $-1 < \eta < 1$ is the correlation coefficient between the in-phase and quadrature scattered waves in each multipath cluster, and the parameters H and h are specified as

$$H = \eta/(1-\eta^2) \qquad \text{and} \qquad h = 1/(1-\eta^2). \quad (4)$$

In our derivations, we apply a widely recognized fact about the validity of decomposition of the $\eta - \mu$ RV into the sum of two gamma RVs [6]. Both gamma RVs are characterized by the same shape parameter μ, and the scale parameters are $\omega_1 = \frac{\Omega_\eta}{2\mu(h+H)}$ and $\omega_2 = \frac{\Omega_\eta}{2\mu(h-H)}$ [10, eq. 4]. Let the scale parameters of the gamma RVs characterizing the SoI components be $\theta_{\mathrm{S}_{1_i}}$ and $\theta_{\mathrm{S}_{2_i}}$ ($i = 1, \cdots N_{\mathrm{S}}$), and the scale parameters of the gamma RVs representing the CCI components be $\theta_{\mathrm{I}_{1_i}}$, and $\theta_{\mathrm{I}_{2_i}}$ ($i = 1, \cdots N_{\mathrm{I}}$).

The PDF of the κ-μ power variable $\gamma_{\kappa-\mu}$, $f_{\gamma_{\kappa-\mu}}$, is given in [6] as

$$f_{\gamma_{\kappa-\mu}}(x) = \frac{\mu(1+\kappa)^{\frac{\mu+1}{2}}x^{\frac{\mu-1}{2}}}{\kappa^{\frac{\mu-1}{2}}\exp(\mu\kappa)\Omega_\kappa^{\frac{\mu+1}{2}}}\exp\left(-\frac{\mu(1+\kappa)x}{\Omega_\kappa}\right) \times I_{\mu-1}\left(2\mu\sqrt{\frac{\kappa(1+\kappa)x}{\Omega_\kappa}}\right) \quad (5)$$

where $\kappa > 0$ is the ratio of the total power of the dominant components to that of the scattered waves, $\Omega_\kappa = E\{\gamma_{\kappa-\mu}\}$, and $\mu = \frac{\Omega_\kappa^2}{2\mathrm{var}\{\gamma_{\kappa-\mu}\}} \cdot \frac{1+2\kappa}{(1+\kappa)^2}$.

In the case of η-μ fading, we assume, without a loss of generality, that the SoI and CCI components in (1) are independent and non-identically distributed (i.n.d.) RVs with the parameters $\{\eta_{\mathrm{S}_i}, \mu_{\eta_{\mathrm{S}_i}}, \Omega_{\eta_{\mathrm{S}_i}}\}$ and $\{\eta_{\mathrm{I}_i}, \mu_{\eta_{\mathrm{I}_i}}, \Omega_{\eta_{\mathrm{I}_i}}\}$, respectively. Obviously, this may not always be the case, and some components may be independent and identically distributed (i.i.d.). In this case, we use the fact that the sum of i.i.d. η-μ RVs is another η-μ RV [6] and merely change the sum of i.i.d. η-μ RVs by a single η-μ RV with properly chosen parameters. In the case of κ-μ fading, we assume the i.i.d. SoI and CCI components, and thus the SoI and CCI are κ-μ–distributed [6]. Let the respective parameters of the SoI and CCI distributions be $\{\kappa_{\mathrm{S}}, \mu_{\kappa_{\mathrm{S}}}, \Omega_{\kappa_{\mathrm{S}}}\}$ and $\{\kappa_{\mathrm{I}}, \mu_{\kappa_{\mathrm{I}}}, \Omega_{\kappa_{\mathrm{I}}}\}$. Thus, the actual number of the SoI and CCI components are included into the model implicitly via the parameters of the statistical distributions.

B. *Evaluation of Cumulative Distribution Function*

By introducing new RVs $X = \sum_{i=1}^{N_{\mathrm{S}}} s_i/\sigma^2$ and $Y = \sum_{i=1}^{N_{\mathrm{I}}} c_i/\sigma^2$, we obtain that the cumulative distribution function (CDF) of γ can be expressed as [4]

$$F_\gamma(z) = \Pr\left\{\frac{X}{Y+1} < z\right\} = \Pr\{X < z(Y+1)\} = \int_0^\infty F_X\left[z\left(y+1\right)\right] f_Y(y)dy \quad (6)$$

where $f_a(z)$ and $F_a(z)$ are the respective PDF and CDF of the RV a.

Then, the following propositions are valid.

Proposition 1: For both η-μ/η-μ and η-μ/κ-μ scenarios, where s_i in (1) are i.n.d. η-μ distributed RVs with integer parameters μ_{S_i}, $i = 1, \cdots N_{\mathrm{S}}$, and c_i are either i.n.d. η-μ– or i.i.d. κ-μ–distributed RVs, the CDF is expressed as

$$F_\gamma(z) = \sum_{k=1}^{2N_{\mathrm{S}}}\sum_{q=1}^{m_k}\frac{\lambda_k^{(m_k-q)}}{\theta_k^{m_k-q}(m_k-q)!} \left[1-\exp\left(-\frac{z}{\theta_k}\right)\sum_{n=0}^{q-1}\frac{z^n}{\theta_k^n n!}\sum_{i=0}^{n}\binom{n}{i}v_{i,k}\right] \quad (7)$$

where $\binom{n}{i}$ is a binomial coefficient, $m_{2l-1} = m_{2l} = \mu_{\mathrm{S}_l}$, $\theta_{2l-1} = \theta_{\mathrm{S}_{1_l}}/\sigma^2$, $\theta_{2l} = \theta_{\mathrm{S}_{2_l}}/\sigma^2$, $l = 1, \cdots, N_{\mathrm{S}}$; $\lambda_k^{(m_k-q)} = \left(\frac{d}{dp}\right)^{m_k-q}\prod_{j=1,j\neq k}^{2N_{\mathrm{S}}}\left(\theta_j^{-1}+p\right)^{-m_j}\Big|_{p=-1/\theta_k} = \sum\frac{(m_k-q)!}{k_1!\ldots k_{2N_{\mathrm{S}}}!}\prod_{j=1,j\neq k}^{2N_{\mathrm{S}}}(-1)^{m_k-q}\frac{(m_j)_{m_k-q}}{(1/\theta_j-1/\theta_k)^{m_j+m_k-q}}$, where the sum is over the indexes $k_1+\ldots+k_{k-1}+k_{k+1}+\ldots k_{2N_{\mathrm{S}}} = (m_k-q)$. To evaluate this expression, we use the Leibniz identity [11, eq. (3.3.8)] and the fact that $\left(\frac{d}{dp}\right)^k\left[(p+a)^{-m_j}\right] = (-1)^k(m_j)_k(p+a)^{-(m_j+k)}$, where $(t)_n$ is the Pochhammer symbol [12, vol. 3, Section II.2].

The parameters $v_{i,k}$ in (7) are defined differently depending on whether the interfering signals are η-μ–faded or κ-μ–faded. For the case of N_{I} i.n.d. η-μ–faded interfering signals, $v_{i,k}$ are defined as

$$v_{i,k}^{\eta-\mu} = \prod_{j=1}^{2N_{\mathrm{I}}}\vartheta_j^{-b_j}\left\{\left(-\frac{d}{dp}\right)^i\prod_{j=1}^{2N_{\mathrm{I}}}\left[(p+1/\vartheta_j)\right]^{-b_j}\right\}\Big|_{p=z/\theta_k} = \prod_{j=1}^{2N_{\mathrm{I}}}\vartheta_j^{-b_j}\sum_{k_1+\ldots+k_{2N_{\mathrm{I}}}=i}\frac{i!}{k_1!\ldots k_{2N_{\mathrm{I}}}!} \times\prod_{j=0}^{2N_{\mathrm{I}}}\frac{(b_j)_{k_j}}{(z/\theta_k+1/\vartheta_j)^{-(b_j+k_j)}} \quad (8)$$

where $b_{2l-1} = b_{2l} = \mu_{\mathrm{I}_l}$, $\vartheta_{2l-1} = \theta_{\mathrm{I}_{1_l}}/\sigma^2$, $\vartheta_{2l} = \theta_{\mathrm{I}_{2_l}}/\sigma^2$, $l = 1, \cdots, N_{\mathrm{I}}$.

For the case of i.i.d. κ-μ–faded interfering signals (that are statistically equivalent to one κ-μ–faded interfering signal), $v_{i,k}$ are defined as

$$
\begin{aligned}
v_{i,k}^{\kappa-\mu} &= \exp(-\alpha)\left(\beta\frac{\sigma^2}{\Omega_{\kappa_{\mathrm{I}}}}\right)^{\mu_{\kappa_{\mathrm{I}}}} \\
&\times\left\{\left(-\frac{d}{dp}\right)^i\left[\frac{\exp\left(\frac{\alpha\cdot\beta\cdot\frac{\sigma^2}{\Omega_{\kappa_{\mathrm{I}}}}}{p+\beta\frac{\sigma^2}{\Omega_{\kappa_{\mathrm{I}}}}}\right)}{\left(p+\beta\frac{\sigma^2}{\Omega_{\kappa_{\mathrm{I}}}}\right)^{\mu_{\kappa_{\mathrm{I}}}}}\right]\right\}\Big|_{p=z/\theta_k} \\
&= i!\left(\beta\frac{\sigma^2}{\Omega_{\kappa_{\mathrm{I}}}}\right)^{\mu_{\kappa_{\mathrm{I}}}}\left(\frac{z}{\theta_k}+\beta\frac{\sigma^2}{\Omega_{\kappa_{\mathrm{I}}}}\right)^{-\mu_{\kappa_{\mathrm{I}}}-i} \\
&\times\exp\left(\frac{-\alpha z}{z+\theta_k\cdot\beta\frac{\sigma^2}{\Omega_{\kappa_{\mathrm{I}}}}}\right)L_i^{\mu_{\kappa_{\mathrm{I}}}-1}\left(\frac{-\alpha\cdot\beta\cdot\sigma^2}{z\frac{\Omega_{\kappa_{\mathrm{I}}}}{\theta_k}+\beta\cdot\sigma^2}\right) \quad (9)
\end{aligned}
$$

where $\alpha = \kappa_{\mathrm{I}}\mu_{\kappa_{\mathrm{I}}}$, $\beta = \mu_{\kappa_{\mathrm{I}}}(1+\kappa_{\mathrm{I}})$, and $L_i^{\nu}(x)$ is the generalized Laguerre polynomial [12], which is a built-in function in many standard software packages.

Proof: See Appendix A.

A recent study [13] proved that the sum of correlated η-μ power RVs with integer or half-integer values of μ_i is statistically equivalent to the sum of independent gamma RVs with properly chosen parameters. In this case, the proof given in Appendix A holds true. Thus, the conditions of proposition 1 can be extended to scenarios where c_i in (1) are correlated η-μ RVs with integer or half-integer values of μ_{I_i}.

If (7)-(9) are used for numerical calculations only, the high-order derivatives in these expressions can simply be evaluated by using the differentiation operator available in many standard software packages, such as *Mathematica* or *Maple*.

Proposition 2: For η-μ/η-μ scenarios with i.n.d. s_i, c_i, and with integer parameters μ_{I_i} of the statistical distributions of c_i, $i=1,\cdots N_{\mathrm{I}}$, the CDF can be expressed as

$$
\begin{aligned}
F_\gamma(z) &= \frac{z^{m_\Sigma}}{\prod_{j=1}^{2N_{\mathrm{I}}}\vartheta_k{}^{b_k}\prod_{l=1}^{2N_{\mathrm{S}}}\theta_l^{m_l}}\sum_{k=1}^{2N_{\mathrm{I}}}\exp\left(\frac{1}{\vartheta_k}\right) \\
&\times\sum_{q=1}^{b_k}\frac{\nu_k^{(b_k-q)}}{(b_k-q)!}\sum_{i=0}^{q-1}\frac{(-1)^{q-i-1}}{(q-1-i)!} \\
&\times\left\{\sum_{k_0+\ldots+k_{2N_{\mathrm{S}}}=i}\frac{\vartheta_k^{1+k_0}}{k_1!\ldots k_{2N_{\mathrm{S}}}!}\prod_{n=1}^{2N_{\mathrm{S}}}\frac{(m_n)_{k_n}}{(1/\vartheta_k+z/\theta_n)^{k_n+m_n}}\right. \\
&+\frac{\exp\left(-\frac{1}{\vartheta_k}\right)}{\Gamma(1+m_\Sigma)}\Phi_2^{(2N_{\mathrm{S}})}\left[(m);1+m_\Sigma;-z\left(\frac{1}{\theta}\right)\right]\sum_{n=0}^{i}\frac{\vartheta_k^{i+1-n}}{n!} \\
&-\sum_{n=0}^{i}\frac{\vartheta_k^{i+1-n}}{\Gamma(1+n+m_\Sigma)}\sum_{k_1+\ldots+k_{2N_{\mathrm{S}}}=n}\prod_{j=1}^{2N_{\mathrm{S}}}\frac{(m_j)_{k_j}}{k_j!} \\
&\left.\times\Phi_2^{(2N_{\mathrm{S}})}\left[(m+k);1+n+m_\Sigma;-z\left(\frac{1}{\theta}\right)+\frac{1}{\vartheta_k}\right]\right\} \quad (10)
\end{aligned}
$$

where (r) and $(r)+v$ denote $r_1,\ldots,r_{2N_{\mathrm{S}}}$ and $r_1+v,\ldots,r_{2N_{\mathrm{S}}}+v$, respectively, $m_\Sigma=\sum_{j=1}^{2N_{\mathrm{S}}}m_j$, $\nu_k^{(b_k-q)} = \left(\frac{d}{dp}\right)^{b_k-q}\prod_{j=1,j\neq k}^{2N_{\mathrm{I}}}\left(\vartheta_j^{-1}+p\right)^{-b_j}\Big|_{p=-1/\vartheta_k}$, and $\Phi_2^{(2N_{\mathrm{S}})}(.)$ is a confluent Lauricella hypergeometric function [14, eq. (10)].

Proof: See Appendix B.

Lauricella's hypergeometric function $\Phi_2^{(2N_{\mathrm{S}})}(.)$ required for the evaluation of (10) is not nowadays available via standard software packages. Its Laplace transform is given in [15, p. 259] as

$$
\begin{aligned}
&\mathcal{L}\left[x^{c-1}\Phi_2^{(2N_{\mathrm{S}})}\left((m);c;-x\left(r\right)\right),\{x,p\}\right] \\
&\quad=\frac{\Gamma(c)}{p^{c-m_\Sigma}}\prod_{l=1}^{2N_{\mathrm{S}}}\left(p+r_l\right)^{-m_l}. \quad (11)
\end{aligned}
$$

Thus, we can write that

$$
\begin{aligned}
&\Phi_2^{(2N_{\mathrm{S}})}\left[(m);c;-(r)\right] \\
&=\frac{\Gamma(c)}{2\pi j}\int_{\chi-j\infty}^{\chi+j\infty}\frac{\exp(p)}{p^{c-m_\Sigma}\prod_{l=1}^{2N_{\mathrm{S}}}\left(p+r_l\right)^{m_l}}dp \quad (12)
\end{aligned}
$$

where χ is a real number that is larger than each pole of the integrand. Then, by reducing the inverse Laplace transform to the inverse Fourier transform by introducing $jw=p-\chi$, we obtain that

$$
\begin{aligned}
\Phi_2^{2N_{\mathrm{S}}}\left[(m);c;-(r)\right] &= \frac{\Gamma(c)\exp(\chi)}{2\pi} \\
\times\int_{-\infty}^{\infty}&\frac{\exp(jw)}{(\chi+jw)^{c-m_\Sigma}\prod_{l=1}^{2N_{\mathrm{S}}}\left(\chi+jw+r_l\right)^{m_l}}dw. \quad (13)
\end{aligned}
$$

The inverse Fourier transform can be efficiently evaluated in *Mathematica* by using the operator NInverseFourierTransform.

Based on the results presented in [13], the conditions of proposition 2 can be extended to scenarios where the SoI components are correlated η-μ RVs with integer or half-integer values of μ_{S_i}.

Proposition 3: Under the κ-μ/η-μ scenario with i.n.d. c_i and with integer parameters μ_{I_i} of all components of CCI, $i=1,\cdots N_{\mathrm{I}}$, the CDF can be expressed as

$$
\begin{aligned}
F_\gamma(z) = 1-\prod_{j=1}^{2N_{\mathrm{I}}}\vartheta_k{}^{-b_k}\sum_{k=1}^{2N_{\mathrm{I}}}\exp\left(\frac{1}{\vartheta_k}\right)\sum_{q=1}^{b_k}\frac{\nu_k^{(b_k-q)}}{(b_k-q)!(q-1)!} \\
\sum_{i=0}^{q-1}\binom{q-1}{i}(-1)^{q-i-1}t_{i,k}. \quad (14)
\end{aligned}
$$

In (14),

$$
\begin{aligned}
t_{0,k} &= \vartheta_k Q_{\mu_{\kappa_{\mathrm{S}}}}\left(a,b\sqrt{z}\right)\exp\left(-\frac{1}{\vartheta_k}\right)-\vartheta_k\left(\frac{b^2z}{b^2z+2/\vartheta_k}\right)^{\mu_{\kappa_{\mathrm{S}}}} \\
&\times\exp\left[-\frac{a^2}{2}+\frac{a^2b^2z}{2(b^2z+2/\vartheta_k)}\right] \\
&\times Q_{\mu_{\kappa_{\mathrm{S}}}}\left[\frac{ab\sqrt{z}}{(b^2z+2/\vartheta_k)^{1/2}},(b^2z+2/\vartheta_k)^{1/2}\right] \quad (15)
\end{aligned}
$$

where $a=\sqrt{2\kappa_{\mathrm{S}}\mu_{\kappa_{\mathrm{S}}}}$, $b=\sigma\sqrt{\frac{2(1+\kappa_{\mathrm{S}})\mu_{\kappa_{\mathrm{S}}}}{\Omega_{\kappa_{\mathrm{S}}}}}$, and $Q_\mu(a,b)=\frac{1}{a^{\mu-1}}\int_b^\infty x^\mu\exp\left(-\frac{x^2+a^2}{2}\right)I_{\mu-1}(ax)dx$ is the generalized Marcum Q function [2, eq. (4.33)]. For $i\geq 1$, $t_{i,k}$ is expressed as

$$t_{i,k} = (-\vartheta_k)^i(-i)_i t_{0,k} + \sum_{j=0}^{i-1}(-1)^{i-j-1}(-i)_{i-j-1}\vartheta_k^{i-j}$$
$$\times\left\{Q_{\mu_{\kappa_{\rm S}}}\left(a, b\sqrt{z}\right)\exp\left(-\frac{1}{\vartheta_k}\right)\right.$$
$$-\frac{(b\sqrt{z})^{\mu_{\kappa_{\rm S}}+1}a^{1-\mu_{\kappa_{\rm S}}}}{(b^2z+2/\vartheta_k)^{j+1+\frac{\mu_{\kappa_{\rm S}}+1}{2}}}$$
$$\times\exp\left[-\frac{a^2}{2}+\frac{a^2b^2z}{2(b^2z+2/\vartheta_k)}\right]$$
$$\left.\times Q_{2j+2+\mu_{\kappa_{\rm S}},\mu_{\kappa_{\rm S}}-1}\left[\frac{ab\sqrt{z}}{(b^2z+2/\vartheta_k)^{1/2}},(b^2z+2/\vartheta_k)^{1/2}\right]\right\}. \tag{16}$$

where $Q_{m,n}(p,q) = \int_q^\infty t^m \exp\left(-\frac{p^2+t^2}{2}\right) I_n(pt)dt$ is the Nuttall function if the parameters m and n are integers, since the function $Q_{m,n}(p,q)$ was originally defined only for integer values of m and n [16, eq. (86)], see also [17]. Otherwise, the function $Q_{m,n}(p,q)$ in (16) can be viewed as an extension of the Nuttall function to real values of m and n.

Proof: See Appendix C.

The Nuttall function is not available via standard software packages, but it can be expressed in terms of the generalized Marcum Q function and modified Bessel function of the first kind implemented in *Mathematica* if $(m-n)$ is an odd integer [17, eq. (8)]. It is easy to show that the derivations in [17] can be directly extended to non-integer values of m and n if $(m-n)$ is a positive odd integer. This is the case in (16). Thus, we can represent the function $Q_{2j+2+\mu_{\kappa_{\rm S}},\mu_{\kappa_{\rm S}}-1}$ in (16) as

$$Q_{2j+2+\mu_{\kappa_{\rm S}},\mu_{\kappa_{\rm S}}-1}(\alpha,\beta) = \sum_{l=1}^{j+2} d_l(j+1)\alpha^{\mu_{\kappa_{\rm S}}-1+2(l-1)}$$
$$\times Q_{\mu_{\kappa_{\rm S}}-1+l}(\alpha,\beta) + \exp\left(-\frac{\alpha^2+\beta^2}{2}\right)$$
$$\times\sum_{l=1}^{j+1} P_{j+1,l}(\beta^2)\alpha^{l-1}\beta^{l+\mu_{\kappa_{\rm S}}} I_{\mu_{\kappa_{\rm S}}+l-2}(\alpha\beta) \tag{17}$$

with

$$d_l(j) = 2^{j-l+1}\binom{j}{j-l+1}\frac{(j-1+\mu_{\kappa_{\rm S}})!}{(\mu_{\kappa_{\rm S}}+l-2)!}$$

and

$$P_{j,l}(\beta^2) = \sum_{n=0}^{j-l} 2^{j-l-n}\binom{j-1-n}{j-l-n}\frac{(\mu_{\kappa_{\rm S}}-1+j)!}{(\mu_{\kappa_{\rm S}}-1+l+n)!}\beta^{2n}.$$

The Nakagami-m and Rayleigh fading models are important special cases of the η-μ distribution [6, Section 3.3.3]. For these particular cases, the derived expressions become simpler. For example, we can obtain from (14)-(15) that under the κ-μ/Rayleigh scenario, the CDF can be expressed only in terms of the Marcum Q function as

$$F_\gamma(z) = 1 - \prod_{i=1}^{N_{\rm I}}(\vartheta_i)^{-1}\sum_{k=1}^{N_{\rm I}}\exp\left(\frac{1}{\vartheta_k}\right)\lambda_k t_{0,k} \tag{18}$$

where $\lambda_k = \prod_{j=1,j\neq k}^{N_{\rm I}}(1/\vartheta_j - 1/\vartheta_k)^{-1}$.

It is seen that the CDF expressions (7), (10), (14), and (18) are given in terms of the ratios $\theta_{{\rm I}_{m_i}}/\sigma^2$, $\theta_{{\rm S}_{m_i}}/\sigma^2$ $(m=1,2)$, $\Omega_{\kappa_{\rm I}}/\sigma^2$, and $b = \sigma\sqrt{\frac{2(1+\kappa_{\rm S})\mu_{\kappa_{\rm S}}}{\Omega_{\kappa_{\rm S}}}}$. We see, however, that these variables can be expressed in terms of the signal-to-noise ratios (SNR), ${\rm SNR}_{\eta_i} = \Omega_{{\rm S}_{\eta_i}}/\sigma^2$ and ${\rm SNR}_\kappa = \Omega_{{\rm S}_\kappa}/\sigma^2$, as well as in terms of the co-channel interference-to-noise ratios (CNR), ${\rm CNR}_{\eta_i} = \Omega_{\eta_{\rm I}}/\sigma^2$ and ${\rm CNR}_\kappa = \Omega_{\kappa_{\rm I}}/\sigma^2$. The parameter $\Omega_{\kappa_{\rm I}}/\sigma^2$ in (9) is just ${\rm CNR}_\kappa$, and the parameter b in (15) is expressed directly in terms of ${\rm SNR}_\kappa$ as

$$b = \sqrt{\frac{2(1+\kappa_{\rm S})\mu_{\kappa_{\rm S}}}{{\rm SNR}_\kappa}}. \tag{19}$$

Expressions for $\theta_{{\rm S}_{m_i}}/\sigma^2$ and $\theta_{{\rm I}_{m_i}}/\sigma^2$, $m=1,2$, in terms of SNR and CNR depend on the format of the η-μ distribution. This is because the parameters of the decomposition of the η-μ power RV into two gamma RVs depend on the format of the η-μ distribution. Based on (3), we find that in format 1,

$$\theta_{{\rm S}_{1_i}}/\sigma^2 = {\rm SNR}_{\eta_i}\frac{\eta_{{\rm S}_i}}{\mu_{\eta_{{\rm S}_i}}(1+\eta_{{\rm S}_i})},$$
$$\theta_{{\rm S}_{2_i}}/\sigma^2 = {\rm SNR}_{\eta_i}\frac{1}{\mu_{\eta_{{\rm S}_i}}(1+\eta_{{\rm S}_i})},$$
$$\theta_{{\rm I}_{1_i}}/\sigma^2 = {\rm CNR}_{\eta_i}\frac{\eta_{{\rm I}_i}}{\mu_{\eta_{{\rm I}_i}}(1+\eta_{{\rm I}_i})},$$
$$\theta_{{\rm I}_{2_i}}/\sigma^2 = {\rm CNR}_{\eta_i}\frac{1}{\mu_{\eta_{{\rm I}_i}}(1+\eta_{{\rm I}_i})}. \tag{20}$$

In format 2, we find from (4) that

$$\theta_{{\rm S}_{1_i}}/\sigma^2 = {\rm SNR}_{\eta_i}\frac{1}{2\mu_{\eta_{{\rm S}_i}}(1-\eta_{{\rm S}_i})},$$
$$\theta_{{\rm S}_{2_i}}/\sigma^2 = {\rm SNR}_{\eta_i}\frac{1}{2\mu_{\eta_{{\rm S}_i}}(1+\eta_{{\rm S}_i})},$$
$$\theta_{{\rm I}_{1_i}}/\sigma^2 = {\rm CNR}_{\eta_i}\frac{1}{2\mu_{\eta_{{\rm I}_i}}(1-\eta_{{\rm I}_i})},$$
$$\theta_{{\rm I}_{2_i}}/\sigma^2 = {\rm CNR}_{\eta_i}\frac{1}{2\mu_{\eta_{{\rm I}_i}}(1+\eta_{{\rm S}_i})}. \tag{21}$$

III. Applications

In this section, we discuss examples of application of the derived theoretical results and present numerical estimates evaluated analytically and via Monte Carlo simulations. In our simulations, we generated the η-μ power RVs as the sum of independent gamma RVs [10, eq. (4)], and we obtained the κ-μ power RVs as the sum of independent gamma and Ricean RVs with properly chosen parameters. This ability follows from the validity of decomposition of the moment generating function (MGF) of the κ-μ distribution [18, eq. (7)] into the product of the gamma and Ricean MGFs. In our evaluations, we assumed format 1 of the η-μ distribution.

The structure of (1) shows that it can be applied, for example, for analyzing the statistical characterization of the SINR in receiver diversity systems that employ maximal ratio combining (MRC) at the receiver. It is known that although MRC is not an optimal strategy under CCI, the method is applied in practical systems as a suboptimal strategy [19]. But it is important to take into account the fact that we obtain the weighted CCI components at the output of the combiner. This fact results in a twofold effect. Firstly, generally the statistical distribution of CCI at the output of the combiner changes. Secondly, generally CCI at the output of the combiner depends on the SoI. The latter is always the case unless CCI is zero-mean Gaussian, that is, unless CCI is Rayleigh–faded [19]. Obviously, these two effects cannot be observed in the case of one receiving antenna.

Model (1) may also be applied for the SINR representation in open-loop transmit diversity (TD) systems.

Numerical results given in this section present estimates of the OP, $P_{\rm out}(\gamma_0) = \Pr\{\gamma \leq \gamma_0\} = F_\gamma(z)\Big|_{z=\gamma_0}$, for different scenarios with $\gamma_0 = 0$ dB. In Fig. 1, we show estimates of the OP for open-loop TD systems with L transmitting antennas operating over η-μ–faded branches with η-μ– and κ-μ–faded CCI. For the scenarios with integer values of $\mu_{{\rm S}_i}$, we applied (7)-(9) for the evaluation, and in the case of integer values of $\mu_{{\rm I}_i}$, we used (10). In Fig. 2, we present estimates of the OP evaluated via (14)-(16) for TD systems with i.i.d. κ-μ–faded SoI components and η-μ–faded CCI components. We assume four interfering signals with the parameters $\{\eta_{{\rm I}_i}, \mu_{{\rm I}_i}, {\rm CNR}_{{\rm I}_i}\}_{i=1}^4 = \{\{0.7; 3; 0{\rm dB}\}, \{0.5; 2; 1{\rm dB}\}, \{0.3; 1; -2{\rm dB}\}, \{0.2; 1; 2{\rm dB}\}\}$. The fading parameters of each propagation path are $\mu = 1.8$, $\kappa = 0.5$, and $\kappa = 2.5$. In both figures, the OP is shown versus the average SINR, which is defined as $\frac{\sum_{i=1}^{N_{\rm S}} SNR_i}{1+\sum_{i=1}^{N_{\rm I}} CNR_i}$.

Under all scenarios, the numerical estimates evaluated analytically and via Monte Carlo simulations agree well.

IV. Conclusion

In this paper, we obtained expressions for the cumulative distribution function of a specially constructed random variable expressed as the ratio of generalized random variables (see (1)). The derived theoretical results can be applied, for example, for evaluating the outage probability in η-μ and κ-μ fading channels with CCI and background noise. More precisely, examples of the application include η-μ/η-μ, η-μ/κ-μ, and κ-μ/η-μ scenarios where the fading parameters μ_i of the η-μ distributions describing the SoI or CCI components are integers. Thus, we extend previously reported results obtained for interference-limited scenarios [8] and for scenarios with background noise, but restricted to the Rice–faded SoI [4] and Rayleigh–faded CCI [4], [9]. For scenarios with integer values of the parameters $\mu_{{\rm S}_i}$ of the η-μ models representing the statistical distributions of the SoI components, we obtained formulas in terms of elementary functions.

The results of this work are restricted to integer values of the parameters μ_i of the η-μ distributions characterizing the SoI or CCI components. Obviously, this may not always be the case in practice. We note, however, that the parameter μ of the η-μ distribution is inversely proportional to the amount of fading under a fixed η [6, eq. (24)]. Thus, the presented expressions may be used as bounds for real estimates of the outage probability for arbitrary values of μ.

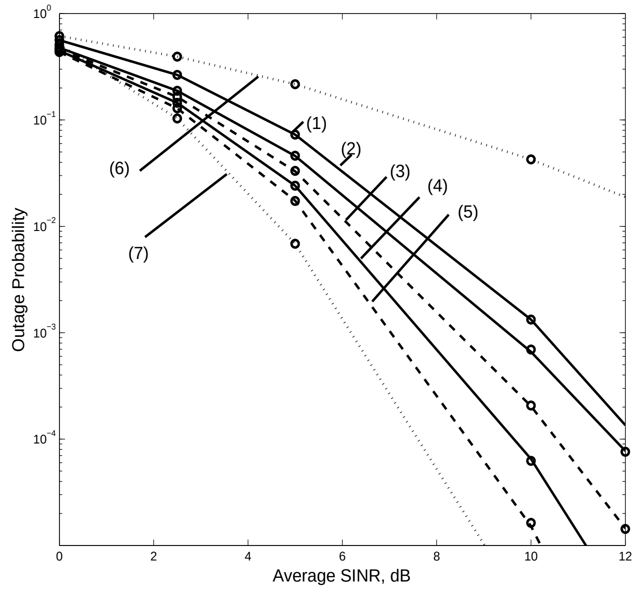

Fig. 1: Outage probability for TD systems under η-μ/η-μ ((1)-(5)) and η-μ/κ-μ ((6)-(7)) scenarios with a different number L of transmitting antennas. Fading parameters are:
(1) - $L = 3$; $\{\eta_{{\rm S}_i}, \mu_{{\rm S}_i}\}_{i=1}^3 = \{\{0.1; 0.8\}, \{0.2; 1.2\}, \{0.3; 1.5\}\}$; $\{\eta_{{\rm I}_i}, \mu_{{\rm I}_i}, {\rm CNR}_{{\rm I}_i}\}_{i=1}^2 = \{\{0.1; 1; 2{\rm dB}\}, \{0.2; 2; 0{\rm dB}\}\}$;
(2)-(5) - two CCI components with $\{\eta_{{\rm I}_i}, \mu_{{\rm I}_i}, {\rm CNR}_{{\rm I}_i}\}_{i=1}^2 = \{\{0.1; 1.2; 5{\rm dB}\}, \{0.2; 1.8; 0{\rm dB}\}\}$;
(2) - $L = 2$; $\{\eta_{{\rm S}_i}, \mu_{{\rm S}_i}\}_{i=1}^2 = \{\{0.6; 1\}, \{0.4; 2\}\}$;
(3) - $L = 3$; $\{\eta_{{\rm S}_i}, \mu_{{\rm S}_i}\}_{i=1}^3 = \{\{0.6; 1\}, \{0.4; 2\}, \{0.4; 1\}\}$;
(4) - $L = 2$; $\{\eta_{{\rm S}_i}, \mu_{{\rm S}_i}\}_{i=1}^2 = \{\{0.6; 2\}, \{0.4; 3\}\}$;
(5) - $L = 3$; $\{\eta_{{\rm S}_i}, \mu_{{\rm S}_i}\}_{i=1}^3 = \{\{0.6; 2\}, \{0.4; 3\}, \{0.4; 2\}\}$;
(6)-(7) - CCI parameters are $\{\kappa_{\rm I} = 1.5; \mu_{\rm I} = 2.3; {\rm CNR} = 0{\rm dB}\}$;
(6) - $L = 1$; $\eta_{\rm S} = 0.1; \mu_{\rm S} = 1$;
(7) - $L = 2$; $\{\eta_{{\rm S}_i}, \mu_{{\rm S}_i}\}_{i=1}^2 = \{\{0.6; 2\}, \{0.4; 3\}\}$.
Single points report simulation results.

Appendix A
Proof of Proposition 1

Based on the validity of decomposition of the η-μ power RV into the sum of two gamma RVs (see, for example, [10, eq. (4)] for details), and taking into account the fact that all $\mu_{{\rm S}_i}$ are integers, we find that the CDF $F_X[z(y+1)]$ can be expressed as

$$F_X\left[z(y+1)\right] = \sum_{k=1}^{2N_{\rm S}} \sum_{q=1}^{m_k} \frac{\lambda_k^{(m_k-q)}}{\theta_k^{m_k-q}(m_k-q)!} \left[1 - \exp\left[-\frac{z(y+1)}{\theta_k}\right] \sum_{n=0}^{q-1} \frac{z^n}{\theta_k^n n!} \sum_{i=0}^{n} \binom{n}{i} y^i\right]. \quad (22)$$

To obtain (22), we use a PDF expression [20, eq. (10)] and finite series expansion of the lower incomplete gamma function $\gamma(q,t)$ for integer values of q, $\gamma(q,t) = (q-1)!\left[1 - \exp(-t)\sum_{n=0}^{q-1} \frac{t^n}{n!}\right]$ [12, vol. 1]. We also apply the binomial theorem for the expansion of $(y+1)^n$ [11]. After plugging (22) into (6), we note that (6) can be expressed as the composition of Laplace transforms $\mathcal{L}\left[y^i f_Y(y), \{y, z/\theta_k\}\right]$,

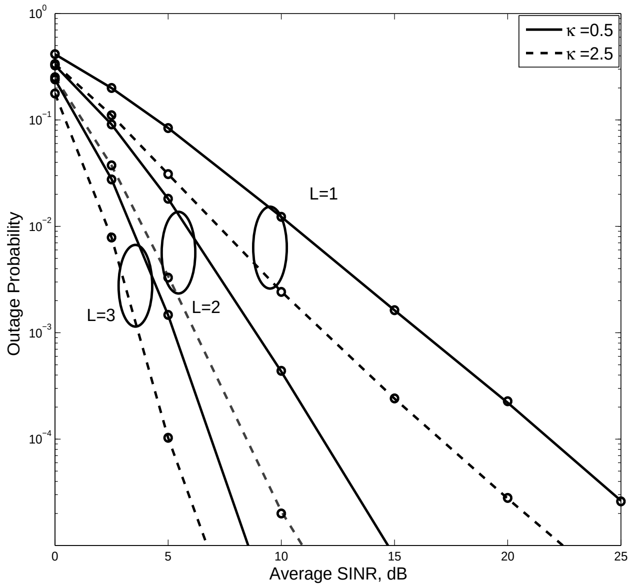

Fig. 2: Outage probability for open-loop TD systems operating in κ-μ fading ($\mu = 1.8$, $\kappa = 0.5$, and $\kappa = 2.5$) with four η-μ–faded interfering signals. CCI parameters are $\{\eta_{\mathrm{I}_i}, \mu_{\mathrm{I}_i}, \mathrm{CNR}_{\mathrm{I}_i}\}_{i=1}^{4} = \{\{0.7; 3; 0\mathrm{dB}\}, \{0.5; 2; 1\mathrm{dB}\}, \{0.3; 1; -2\mathrm{dB}\}, \{0.2; 1; 2\mathrm{dB}\}\}$. Single points report simulation results.

where i is an integer. Thus we obtain that

$$F_\gamma(z) = \sum_{k=1}^{2N_\mathrm{S}} \sum_{q=1}^{m_i} \frac{\lambda_k^{(m_k-q)} \theta_k^q}{\theta_k^{m_k-q}(m_k-q)!} \left\{ 1 - \exp\left(-\frac{z}{\theta_k}\right) \right. \times \sum_{n=0}^{q-1} \frac{z^n}{\theta_k^n n!} \sum_{i=0}^{n} \binom{n}{i} \left. \times \left[\left(-\frac{d}{dp}\right)^i \mathcal{L}\left[f_Y(y), \{y,p\}\right] \right] \Big|_{p=z/\theta_k} \right\} \tag{23}$$

where we use the fact that $\mathcal{L}\left[y^i f_Y(y), \{y,p\}\right] = \left(-\frac{d}{dp}\right)^i \mathcal{L}\left[f_Y(y), \{y,p\}\right]$ if i is an integer [12, vol. 4, eq. (1.1.2.9)].

Then, by taking into account expressions for the MGFs of the η-μ and κ-μ distributions [18, eqs. (6)-(7)] as well as the fact that the MGF of the sum of independent components is the product of the individual MGFs, we immediately obtain (8) and (9) by using Leibniz's identity and a differentiation formula [21, eq. (1.1.3.2)]. ■

Appendix B
Proof of Proposition 2

We represent (6) in an equivalent form as

$$F_\gamma(z) = \int_1^\infty F_X(zy) f_Y(y-1) dy. \tag{24}$$

and take into account that [20, eq. (10)]

$$f_Y(y-1) = \prod_{j=1}^{2N_\mathrm{I}} \vartheta_k^{-b_k} \sum_{k=1}^{2N_\mathrm{I}} \exp\left(\frac{1}{\vartheta_k}\right) \sum_{q=1}^{b_k} \frac{\nu_k^{(b_k-q)}}{(b_k-q)!(q-1)!} \times \sum_{i=0}^{q-1} \binom{q-1}{i} (-1)^{q-i-1} y^i \exp\left(-\frac{y}{\vartheta_k}\right) \tag{25}$$

as well as that [14, eq. (13)]

$$F_X(zy) = \frac{(zy)^{m_\Sigma}}{\prod_{j=1}^{2N_\mathrm{S}} \theta_j^{m_j} \Gamma(1+m_\Sigma)} \times \Phi_2^{(2N_\mathrm{S})}\left(m_1, \ldots, m_{2N_\mathrm{S}}; 1+m_\Sigma; -\frac{zy}{\theta_1}, \ldots, -\frac{zy}{\theta_{2N_\mathrm{S}}}\right). \tag{26}$$

Then we obtain from (24)-(25) that

$$F_\gamma(z) = \prod_{j=1}^{2N_\mathrm{I}} \vartheta_k^{-b_k} \sum_{k=1}^{2N_\mathrm{I}} \exp\left(\frac{1}{\vartheta_k}\right) \sum_{q=1}^{b_k} \frac{\nu_k^{(b_k-q)}}{(b_k-q)!(q-1)!} \times \sum_{i=0}^{q-1} \binom{q-1}{i} (-1)^{q-i-1} \left\{ \underbrace{\int_0^\infty y^i \exp\left(-\frac{y}{\vartheta_k}\right) F_X(zy) dy}_{I_1} - \underbrace{\int_0^1 y^i \exp\left(-\frac{y}{\vartheta_k}\right) F_X(zy) dy}_{I_2} \right\}. \tag{27}$$

It is seen that

$$I_1 = \mathcal{L}\left[y^i F_X(zy), \{y,p\}\right] \Big|_{p=1/\vartheta_k} = \frac{1}{z} \times \left(-\frac{d}{dp}\right)^i \mathcal{L}\left[F_X(y), \{y,p/z\}\right] \Big|_{p=1/\vartheta_k} = \frac{z^{m_\Sigma}}{\prod_{j=1}^{2N_\mathrm{S}} \theta_j^{m_j}} \times \sum_{k_0+\ldots+k_{2N_\mathrm{S}}=i} \frac{\vartheta_k^{1+k_0}}{k_1! \ldots k_{2N_\mathrm{S}}!} \times \prod_{n=1}^{2N_\mathrm{S}} \frac{(m_n)_{k_n}}{(1/\vartheta_k + z/\theta_n)^{k_n+m_n}}. \tag{28}$$

To obtain (28), we use (26) and (11), as well as apply Leibniz's identity.

The integral I_2 in (27) can be evaluated by parts. Doing this, we ascertain that

$$I_2 = \vartheta_k^{i+1} i! \sum_{n=0}^{i} \frac{1}{n! \vartheta_k^n} \left[-\exp\left(-\frac{1}{\vartheta_k}\right) F_X(z) + \mathcal{L}^{-1}\left\{ \frac{1}{p}\left(-\frac{d}{dp}\right)^n \times \prod_{j=1}^{2N_\mathrm{S}} [1 + (p+1/\vartheta_k)\theta_j/z]^{-m_j}; p, x \right\} \Big|_{x=1} \right] \tag{29}$$

wherein we use Laplace transform properties [12, vol. 4, eqs. (1.1.2.1), (1.1.2.9), (1.1.3.1), and (1.1.5.3)]. We find from (29) that

$$I_2 = \frac{z^{m_\Sigma}}{\prod_{j=1}^{2N_{\rm S}} \theta_j^{m_j}} \left\{ -\frac{\exp\left(-\frac{1}{\vartheta_k}\right)}{\Gamma\left(1+m_\Sigma\right)} \sum_{n=0}^{i} \frac{\vartheta_k^{i+1-n}}{n!} \right.$$
$$\times \Phi_2^{(2N_{\rm S})}\left(m_1,\ldots,m_{2N_{\rm S}};1+m_\Sigma;-\frac{z}{\theta_1},\ldots,-\frac{z}{\theta_{2N_{\rm S}}}\right)$$
$$+\sum_{n=0}^{i}\frac{\vartheta_k^{i+1-n}}{\Gamma(1+n+m_\Sigma)}\sum_{k_1+\ldots+k_{2N_{\rm S}}=n}\prod_{j=1}^{2N_{\rm S}}\frac{(m_j)_{k_j}}{k_j!}$$
$$\times\Phi_2^{(2N_{\rm S})}\left[m_1+k_1,\ldots,m_{2N_{\rm S}}+k_{2N_{\rm S}};1+n+m_\Sigma;\right.$$
$$\left.\left.-\left(\frac{z}{\theta_i}+\frac{1}{\vartheta_k}\right),\ldots,-\left(\frac{z}{\theta_{2N_{\rm S}}}+\frac{1}{\vartheta_k}\right)\right]\right\}. \tag{30}$$

To obtain (30), we use (26) and (11), as well as apply Leibniz's identity. ■

Appendix C
Proof of Proposition 3

We note that the evaluation of (24) with $f_Y(y-1)$ defined by (25) and $F_X(zy)$ representing the CDF of the κ-μ distribution as [6, eq. (3)]

$$F_X(zy) = 1 - Q_{\mu_{\kappa_{\rm S}}}\left[\sqrt{2\kappa_{\rm S}\mu_{\kappa_{\rm S}}}, \sigma\sqrt{\frac{2(1+\kappa_{\rm S})\mu_{\kappa_{\rm S}}zy}{\Omega_{\kappa_{\rm S}}}}\right], \tag{31}$$

reduces to the evaluation of the integrals of the form

$$t_{i,k} = \int_1^\infty y^i \exp\left(-\frac{y}{\vartheta_k}\right)$$
$$\times Q_{\mu_{\kappa_{\rm S}}}\left[\sqrt{2\kappa_{\rm S}\mu_{\kappa_{\rm S}}}, \sigma\sqrt{\frac{2(1+\kappa_{\rm S})\mu_{\kappa_{\rm S}}zy}{\Omega_{\kappa_{\rm S}}}}\right] dy. \tag{32}$$

Let $a = \sqrt{2\kappa_{\rm S}\mu_{\kappa_{\rm S}}}$ and $b = \sigma\sqrt{\frac{2(1+\kappa_{\rm S})\mu_{\kappa_{\rm S}}}{\Omega_{\kappa_{\rm S}}}}$. By taking into account the fact that [2, eq. (4.33)]

$$\frac{d}{dy}Q_{\mu_{\kappa_{\rm S}}}\left[a, b\sqrt{zy}\right] = -\frac{(b\sqrt{z})^{\mu_{\kappa_{\rm S}}+1}}{2a^{\mu_{\kappa_{\rm S}}-1}} y^{\frac{\mu_{\kappa_{\rm S}}-1}{2}}$$
$$\times \exp\left(-\frac{a^2+b^2zy}{2}\right) I_{\mu_{\kappa_{\rm S}}-1}\left(ab\sqrt{zy}\right)$$

and evaluating (32) by parts, we obtain a recurrence equation for $t_{i,k}, i \geq 1$ as

$$t_{i,k} = i\vartheta_k t_{i-1,k} + \vartheta_k Q_{\mu_{\kappa_{\rm S}}}\left(a, b\sqrt{z}\right)\exp\left(-\frac{1}{\vartheta_k}\right)$$
$$-\vartheta_k \frac{(b\sqrt{z})^{\mu_{\kappa_{\rm S}}+1}a^{1-\mu_{\kappa_{\rm S}}}}{(b^2z+2/\vartheta_k)^{i+\frac{\mu_{\kappa_{\rm S}}+1}{2}}}\exp\left[-\frac{a^2}{2}+\frac{a^2b^2z}{2(b^2z+2/\vartheta_k)}\right]$$
$$\times Q_{2i+\mu_{\kappa_{\rm S}},\mu_{\kappa_{\rm S}}-1}\left[\frac{ab\sqrt{z}}{(b^2z+2/\vartheta_k)^{1/2}},\right.$$
$$\left.(b^2z+2/\vartheta_k)^{1/2}\right]. \tag{33}$$

Eq. (15) is obtained by evaluation of (32) by parts for $i = 0$, and (16) is derived on the basis of (33). ■

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