Incorporating Stiefel Geometry in Codebook Design and Selection for Improved Base Station Cooperation

Renaud-Alexandre Pitaval and Olav Tirkkonen
Aalto University, Department of Communications and Networking, Espoo, Finland
email: {renaud-alexandre.pitaval, olav.tirkkonen}@aalto.fi

Abstract—**Base station cooperation is expected to enhance spectrum efficiency of future cellular system. Performance heavily depends on the channel state information available at the transmitter. In practical systems, channel information are acquire through a limited feedback channel. Typically, quantization of the channel at the receiver side is done with a fixed pre-designed codebook. In this paper, we consider the codebook design and codeword selection problem when a product codebook is employed, reusing a point-to-point codebook. Point-to-point codebooks are often designed as Grassmannian packings. To improve the performance of the codebook for base station cooperation without impairing the performance for single cell transmission, we propose a novel joint Grassmann-Stiefel codebook design. In addition, we propose a method for independently selecting the per-cell codewords by using a distance on the Stiefel manifold.**

I. Introduction

Base station (BS) cooperation or coordinated multi-point (CoMP) is expected to be a key technology in future wireless cellular systems [1]. In order to synchronize the transmission between the cooperative BSs, the receiver needs to feed back channel state information (CSI) to the transmitters. In frequency-division duplex systems, the only way to acquire CSI is through a limited feedback channel. A widely applied method is to use codebook-based precoding in which the receiver selects a precoding codeword from a predefined codebook and feeds back the index to the transmitter. Since it is more important to feed back the channel direction than the channel beam gain [2], the quantization of the eigendirections of the channel is often done with a unitary code.

In point-to-point communications, the performance of a unitary precoding codebook depends of the distance properties of the Grassmannian planes generated by the codebook. This has led to the well known Grassmannian codebook design [3] where the codebook is understood as a discretization of the Grassmann manifold. Since a Grassmannian codeword represents a subspace and an infinite number of different orthonormal matrices span this same subspace, a suitable representative in this equivalence class has to be chosen. It follows that the chosen representative does not impact the performance in point-to-point MIMO communications.

The work of Renaud-Alexandre Pitaval has been supported in part by the Academy of Finland, and Tekes - the Finnish Funding Agency for Technology and Innovation

In cooperative MIMO BS, the receiver has to quantize an aggregate channel matrix carrying the transmission from the multiple BSs. Preliminary work considered transposing point-to-point limited feedback design to cooperative systems [1], [4]. However, as described in [5], the aggregate channel matrix is a specific feature that cannot be addressed by directly applying point-to-point codebook based precoding. Depending on the user position in the cell, the number of cooperative BS as well as the large scale path loss from the BSs would vary and thus be dynamic. It may also be desirable to allow the network to choose the active BSs for a transmission, independently of the feedback.

As a consequence, the authors of [5] proposed to employ a product codebook constructed by concatenating codewords of a single cell codebook. The method is flexible and has the evident advantage that it reuses point-to-point codebooks. To decrease codeword selection complexity, it was subsequently suggested in [6] that the receiver quantizes independently each per-cell channel. While this decreases complexity, it induces a loss in performance. This loss is a consequence of dismissing the phase ambiguity between per-cell channels, as recognized in [6], [7], [8]. To solve this problem the authors of [6], [7], [8] suggested that the receiver feeds back additional bits related to this phase ambiguity.

Since matrix concatenation is employed in the product codebook, it is not enough to quantize the channel subspace of the per-cell codebook. Instead, quantizing the full space of unitary precoders, the Stiefel manifold, could be considered. Nevertheless, in practical system it would be desirable that a single codebook can be employed for Grassmann and Stiefel quantization. In this paper we show that the representative of a per-cell Grassmannian codebook can be chosen appropriately to yield a better product codebook. This leads to a new codebook design problem, where first one has to design a Grassmannian codebook, then one chooses the representative for every codeword to efficiently quantize the Stiefel manifold.

In addition, we give an independent codeword selection method to take into account the phase ambiguity between the per-cell channel that does not require any additional information to feed back. This is done by applying a distance on the Stiefel manifold for codeword selection rather than the typical Grassmannian chordal distance.

This paper is organized as follows. Section II defines the pertinent mathematical spaces for the codebook design. In Section III, the system model and the codebook design problem are presented. In Section IV, we present a joint Grassmann-Stiefel codebook design and illustrated it with a toy example. In section V are presented different codeword selection principles, from which a independent selection using the Stiefel distance. Section VI presents simulation results, and finally Section VII concludes.

II. Definitions

The primary codebooks we considered are designed for transmission of n_s-streams from a n_t-antenna base station. Pertinent spaces for the codebook design are as follows.

Unitary Group: The codebooks addressed have orthonormal columns, and consist thus of a number of columns from a unitary matrix. The space of all n_t-dimensional unitary matrices is denoted by the unitary group:

$$\mathcal{U}_{n_t} = \left\{ \mathbf{U} \in \mathbb{C}^{n_t \times n_t} \quad | \quad \mathbf{U}^H \mathbf{U} = \mathbf{I}_{n_t} \right\}.$$

Stiefel manifold: The complex Stiefel manifold $\mathcal{V}^{\mathbb{C}}_{n_t,n_s}$ is defined as the space of orthonormal rectangular matrices (with $n_s \leq n_t$):

$$\mathcal{V}^{\mathbb{C}}_{n_t,n_s} = \left\{ \mathbf{Y} \in \mathbb{C}^{n_t \times n_s} \quad | \quad \mathbf{Y}^H \mathbf{Y} = \mathbf{I}_{n_s} \right\}. \tag{1}$$

When $n_s = 1$, the Stiefel manifold is the set of unit vectors in $\mathbb{C}^{n_t}$ which can be identified as a hypersphere in $\mathbb{R}^{2n_t}$. Otherwise, for general values of n_s, the Stiefel manifold is a subspace of a hypersphere in $\mathbb{R}^{2n_t n_s}$. The standard distance considered on the Stiefel manifold is thus

$$d_s(\mathbf{X}, \mathbf{Y}) = \|\mathbf{X} - \mathbf{Y}\|_F \tag{2}$$

$$= \sqrt{2n_s - 2\Re(\mathrm{Tr}[\mathbf{X}^H \mathbf{Y}])} \tag{3}$$

Grassmann manifold: The complex Grassmann manifold $\mathcal{G}^{\mathbb{C}}_{n_t,n_s}$ is the set of all n_s-dimensional subspaces of $\mathbb{C}^{n_t}$. $\mathcal{G}^{\mathbb{C}}_{n_t,n_s}$ can be expressed as the quotient space of the Stiefel manifold and the unitary group: $\mathcal{G}^{\mathbb{C}}_{n_t,n_s} \cong \mathcal{V}^{\mathbb{C}}_{n_t,n_s}/\mathcal{U}_{n_s}$. A point in the Grassmann manifold can thus be represented as the equivalence class of the $n_t \times n_s$ orthonormal matrices whose columns span the same space:

$$[\mathbf{Y}] = \{\mathbf{Y}\mathbf{U} \quad | \quad \mathbf{U} \in \mathcal{U}_{n_s}\}. \tag{4}$$

where the Stiefel-matrix $\mathbf{Y}$ is an $n_t \times n_s$ matrix with orthonormal columns. Each column in $\mathbf{Y}$ determines a line in $\mathbb{C}^{n_t}$, so that each $\mathbf{Y}$ determines an n_s-dimensional subspace in $\mathbb{C}^{n_t}$. The equivalence class $[\mathbf{Y}]$ thus represents a set of matrices $\mathbf{Y}$ determining the same subspace. Taking two Grassmannian points $[\mathbf{X}]$, $[\mathbf{Y}] \in \mathcal{G}^{\mathbb{C}}_{n_t,n_s}$, the representatives $\mathbf{X}$, $\mathbf{Y} \in \mathcal{V}^{\mathbb{C}}_{n_t,n_s}$ determine two subspaces of $\mathbb{C}^{n_t}$. The chordal distance between these is defined as [9]

$$d_g([\mathbf{X}], [\mathbf{Y}]) = \frac{1}{\sqrt{2}} \|\mathbf{X}\mathbf{X}^H - \mathbf{Y}\mathbf{Y}^H\|_F \tag{5}$$

$$= \sqrt{n_s - \|\mathbf{X}^H \mathbf{Y}\|_F^2} \tag{6}$$

This distance does not depend on the representative in $[\mathbf{X}]$ and $[\mathbf{Y}]$ chosen.

Stiefel and Grassmannian codebook: A code or a codebook is a finite subset of points in the considered space. Since a Grassmannian codebook is a set of equivalence classes, it may be represented by a suitable representative in each equivalence class. The obtained set of rectangular unitary matrices is inherently both a Grassmannian code and a Stiefel code. The distance properties of the code in these two interpretations depend on the design principle.

III. System Model

We consider precoded transmission over a cooperative MIMO multi-cell system with n_{bs} base stations each equipped with n_t antennas. It is assumed that the BSs are able to instantaneously share the feedback information, e.g. via high speed backhauls. When the BSs transmit to a user, the received signal is

$$\mathbf{y} = \mathbf{H}_{\text{ls}} \mathbf{W} \mathbf{x} + \mathbf{n}, \tag{7}$$

where $\mathbf{y} \in \mathbb{C}^{n_r \times 1}$ is the received vector, $\mathbf{W}$ is an $n_{bs} n_t \times n_s$ aggregate precoding matrix, $\mathbf{x}$ is an $n_s \times 1$ vector of information symbols, n_s is the number of streams, and

$$\mathbf{H}_{\text{ls}} = [\alpha_1 \mathbf{H}_1, \ldots, \alpha_{n_{bs}} \mathbf{H}_{n_{bs}}] = \mathbf{H}_{\text{ss}} \mathbf{G} \tag{8}$$

is the aggregate channel matrix where the channels from the BSs to the receiver are concatenated, and large scale path losses are explicitly taken into account. The average path gain from the ith BS to the receiver is α_i, incorporating distance-dependent path loss and shadowing. Small-scale path gains are characterized by the matrices $\mathbf{H}_i \in \mathbb{C}^{n_r \times n_t}$. The aggregate small-scale path gain matrix is denoted $\mathbf{H}_{\text{ss}} = [\mathbf{H}_1, \ldots, \mathbf{H}_{n_{bs}}]$ and the large scale path gains by $\mathbf{G} = \mathrm{diag}(\alpha_1 \mathbf{I}_{n_t}, \ldots, \alpha_{n_{bs}} \mathbf{I}_{n_t})$.

We concentrate on designing $\mathbf{W}$ which steers the transmitted energy to the signal subspace of the receiver. We assume that the total transmit power is $\mathrm{Tr}[\mathbf{W}^H \mathbf{W}] = n_s$, which is equally shared among the symbols such that the diagonal entries $[\mathbf{W}^H \mathbf{W}]_{ii} = 1$. We denote by $\mathbf{V}_{\text{ls}}$ and $\mathbf{V}_{\text{ss}}$ the right singular vectors associated with the n_s largest singular values of $\mathbf{H}_{\text{ls}}$ and $\mathbf{H}_{\text{ss}}$, respectively. The optimum precoding matrix is then $\mathbf{W}_{\text{opt}} = \mathbf{V}_{\text{ls}}$.

The optimum Grassmannian codeword can be written without loss of generality in terms of component codewords as $\mathbf{W}_{\text{opt}} = [\mathbf{W}^H_{\text{opt},1}, \ldots, \mathbf{W}^H_{\text{opt},n_{bs}}]^H$ where $\mathbf{W}_{\text{opt},i} \in \mathbb{C}^{n_t \times n_s}$. The n_s columns of $\mathbf{W}_{\text{opt}}$ are orthogonal, and any codeword that is achieved by multiplying with a $n_s \times n_s$ unitary matrix from the right is equivalent. This means that of its components, $\mathbf{W}_{\text{opt},1}$ could be any $n_t \times n_s$ matrix up to the right unitary rotation, whereas $\mathbf{W}_{\text{opt},i}$ $i > 1$ could be any $n_t \times n_s$ matrix. If quantizing $\mathbf{W} = \mathcal{Q}(\mathbf{W}_{\text{opt}})$ with a per BS power constraint, we would have the full space of normed rectangular matrices, for all except one BS, and the full space of normed matrices modulo unitary rotations for the remaining one. The normalization can be per $\mathbf{W}_i$, or for example per column of $\mathbf{W}_i$, or per row of $\mathbf{W}_i$, or per element on $\mathbf{W}_i$, depending on the wish. In the case of $n_s = 1$ this would mean that we would have Grassmannian degrees of freedom (d.o.f) for one BS and

Stiefel d.o.f. for the rest. For $n_s > 1$ it is remarkable that the per-BS codewords $\mathbf{W}_{\text{opt},i}$ do not necessarily have orthogonal columns.

In order to accommodate to the possible dynamic number of cooperating BSs and deal with the heterogeneous path loss effects, it has been proposed that the receiver quantizes $\mathbf{V}_{\text{ss}}$ rather than $\mathbf{V}_{\text{ls}}$ directly by reusing point-to-point codebooks [5]. The receiver uses a single pre-design codebook $\mathcal{C} = \{\mathbf{C}_1, \ldots, \mathbf{C}_N\}$ of $(n_t \times n_s)$-orthonormal matrices to quantize the signal subspace seen at the receiver. The point-to-point codebook is fixed and independent of the number of cooperating BSs and large-scale path loss effects. Based on this principle, [5] proposes a product codebook which is a direct product of single-cell Grassmannian codebooks, $\mathcal{C}_{pr} = \frac{1}{\sqrt{n_{bs}}}\mathcal{C} \otimes \cdots \otimes \mathcal{C}$: a codeword in $\mathcal{C}_{pr}$ is a normalized concatenation of n_{bs} single cell codewords. This has the benefit that the network can select the BSs that transmit independently of the feedback. It has two non-idealities. First, the Grassmannian codebooks are not necessarily the best ones for Stiefel quantization. Indeed, in [6], a phase ambiguity problem is proposed. Second, even in generic direct product codebooks a minor loss would arise from the fact that joint discretization would allow a somewhat better minimum distance property.

IV. Joint Grassmann-Stiefel codebook

We follow the principles of [5] where the same codebook is used to feedback to all BSs, and that any selection of BSs can be used to transmit to the user. Hence we require orthogonality of the columns in each $\mathbf{W}_i$. With a direct product codebook, the eigendirection of the channel is quantized by concatenating codewords. Thus the choice of representative for a Grassmannian codeword will impact cooperative transmission performance. We propose that the codebook is constructed by first designing a Grassmannian codebook according to standard criteria such as maximizing the minimum distance or minimizing the average distortion. Then, the representative in each Grassmannian plane in the codebook is chosen to optimize a metric on the Stiefel manifold. This means that we select a good Stiefel codebook conditioned on the codebook being simultaneously a near-optimal Grassmannian codebook. In addition to improving performance, this joint Grassmann-Stiefel codebook design has the flexibility that only a single codebook has to be implemented for the different communications scenarios.

To illustrate the joint Grassmann-Stiefel codebook design problem, we consider the toy scenario of building a real codebook of four codewords for a transmission from 3 antennas. This leads to a rare example where visualization of the proposed approach is possible. The real Grassmannian $\mathcal{G}^{\mathbb{R}}_{3,1}$ that needs to be discretized is the set of lines through the origin in the 3D Euclidean space. It can be understood as the set of of antipodal points on the real unit sphere. The corresponding Stiefel manifold is the space of all 3D unit-norm vectors, and can be understood as the full sphere. A Grassmannian code is then a set of antipodal points, and choosing a representative for every Grassmannian codeword means simply choosing one of the two antipodal points on the sphere. A Stiefel-codebook, in turn, is a spherical code. The best four-codeword Grassmannian packing is found by taking the vertices of a cube – the eight vertices of the cube consist of four pairs of antipodal points, i.e. four Grassmannian lines. From this cube, there is four possible non-equivalent four-codeword spherical codes: for example by taking only points in the upper hemisphere we get a square, or by taking two points in both upper and lower hemispheres we get a tetrahedron as depicted on Fig 1. The best Grassmann-Stiefel codebooks is obtained by taking the vertices of the cube that form a tetrahedron. It turns out that the vertices of the tetrahedon gives actually the optimum 4-point spherical (Stiefel) codes under several criteria [10]. In this simple example, it is thus possible to have a codebook that is simultaneously an optimal Grassmannian and Stiefel packing.

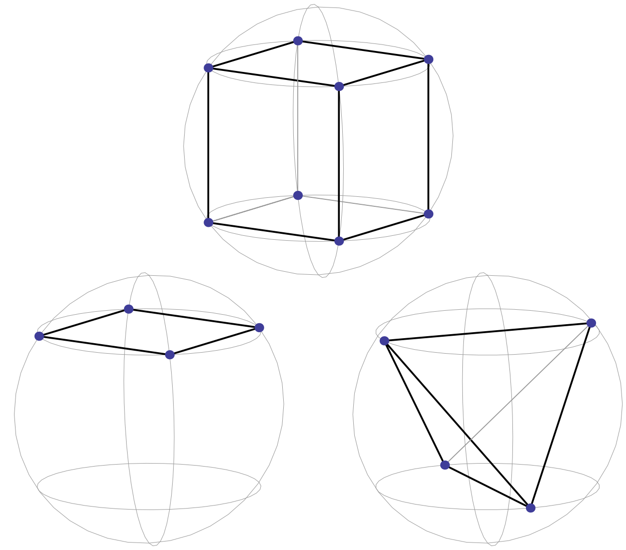

Fig. 1. Illustration of the joint Grassmannian-Stiefel codebook design. On the upper graph the optimum 2-bit Grassmannian packing in $\mathcal{G}^{\mathbb{R}}_{3,1}$, a set of 4 antipodal points forming a cube. On the lower part, two alternatives of 2-bit Stiefel codebooks generating the above Grassmannian codebook: a square and a tetrahedron.

V. Codeword selection

The codeword of the joint codebook can be selected either jointly [5], [6] or independently [6] over the components of $\mathbf{V}_{\text{ss}} = [\mathbf{V}^H_{\text{ss},1}, \ldots, \mathbf{V}^H_{\text{ss},n_{bs}}]^H$. Here an improved independent codeword selection using Stiefel distance is presented. For non-unitary matrices the distances d_s and d_g are defined as (2), (5) respectively.

a) Joint codeword selection: In joint selection [5], the product codebook codeword minimizing

$$\mathbf{W}_{\text{ss}} = \mathcal{Q}_{js}(\mathbf{V}_{\text{ss}}) = \arg\min_{\mathbf{C}_i \in \mathcal{C}_{pr}} d_g(\mathbf{C}_i, \mathbf{V}_{\text{ss}}) \tag{9}$$

is selected. This leads to high complexity due to the size of the exhaustive search required [5], [6].

b) Joint codeword selection with transformed codebook: Joint selection can be improved [6] by borrowing the idea of transformed codebook for spatially correlated channel [11]:

$$\mathbf{W}_{\text{ss}} = \mathcal{Q}_{js-trans}(\mathbf{V}_{\text{ls}}) = \arg\min_{\mathbf{C}_i \in \mathcal{C}_{pr}} d_g(\mathbf{G}\mathbf{C}_i, \mathbf{V}_{\text{ls}}). \tag{10}$$

It is worth noticing that $\frac{\sqrt{n_t n_{bs}}}{\|\mathbf{G}\|}\mathbf{G}\mathbf{C}_i \in \mathcal{V}^{\mathbb{C}}_{n_t n_{bs}, n_s}$. The two joint selection methods provide similar performance for cell edge user, where $\mathbf{G} \propto \mathbf{I}$.

c) Independent codeword selection: As an alternative, each single cell channel matrix could be quantized independently [6]:

$$\mathbf{W}_{\mathrm{ss},k} = \mathcal{Q}_{ind}(\mathbf{V}_{\mathrm{ss},k}) = \arg\min_{\mathbf{C}_i \in \mathcal{C}} d_g(\mathbf{C}_i, \mathbf{V}_{\mathrm{ss},k}) \tag{11}$$

This method leads to a loss of performance as it does not take into account the phase ambiguity between the components of the optimum precoding vector as recognized in [6].

d) Independent codeword selection with Stiefel distance: In order to independently quantize the channel efficiently, the phase ambiguity between the different channels should be taken into account. We suggest that first the strongest channel (with the largest α_i) is quantized using the chordal distance

$$\mathbf{W}_{\mathrm{ss},1} = \mathcal{Q}_{ind}(\mathbf{V}_{\mathrm{ss},1}) = \arg\min_{\mathbf{C}_i \in \mathcal{C}} d_g(\mathbf{C}_i, \mathbf{V}_{\mathrm{ss},1}) \ . \tag{12}$$

The unitary rotation not seen by this Grassmannian codeword selection can be found by performing the polar decomposition $\mathbf{W}_1^H \mathbf{V}_{\mathrm{ss},1} = \mathbf{P}\mathbf{R}$ where $\mathbf{R} \in \mathcal{U}_{n_s}$ and $\mathbf{P}$ is a positive-semidefinite Hermitian matrix. The channels from the other BSs, with the rotation $\mathbf{R}$ removed, are then discretized using the Stiefel distance:

$$\mathbf{W}_{\mathrm{ss},k} = \mathcal{Q}_{stief}(\mathbf{V}_{\mathrm{ss},k}) = \arg\min_{\mathbf{C}_i \in \mathcal{C}} d_s(\mathbf{C}_i, \mathbf{V}_{\mathrm{ss},k}\mathbf{R}^H) \tag{13}$$

To clarify the proposed codeword selection, first, we discuss single stream transmission. The corresponding joint codeword selection is to minimize $d_g(\mathbf{c}_i, \mathbf{v}_{\mathrm{ss}})$ over the possible codeword indexed by i which is equivalent to maximizing

$$\begin{aligned} \left|\mathbf{c}_i^H \mathbf{v}_{\mathrm{ss}}\right| &= \left|\sum_{k=1}^{n_{bs}} \mathbf{c}_{i,k}^H \mathbf{v}_{\mathrm{ss},k}\right| = \left|\sum_{k=1}^{n_{bs}} |\mathbf{c}_{i,k}^H \mathbf{v}_{\mathrm{ss},k}| e^{i\phi_k}\right| && (14) \\ &= \left| |\mathbf{c}_{i,1}^H \mathbf{v}_{\mathrm{ss},1}| + \sum_{k=2}^{n_{bs}} |\mathbf{c}_{i,k}^H \mathbf{v}_{\mathrm{ss},k}| e^{i(\phi_k - \phi_1)} \right| && (15) \\ &= \left| |\mathbf{c}_{i,1}^H \mathbf{v}_{\mathrm{ss},1}| + \sum_{k=2}^{n_{bs}} \mathbf{c}_{i,k}^H (\mathbf{v}_{\mathrm{ss},k} e^{-i\phi_1}) \right| . && (16) \end{aligned}$$

The loss of independent quantization of the $|\mathbf{c}_{i,k}^H \mathbf{v}_{\mathrm{ss},k}|$ is due to the lack of catching the phase ambiguity $e^{i(\phi_k - \phi_1)}$ in the process. This can be solved by using a distance on the Stiefel manifold for quantizing $\mathbf{v}_{\mathrm{ss},k}\mathbf{r}^H$ for $k \neq 1$ where $\mathbf{r} = e^{i\phi_1} = \frac{\mathbf{c}_{i,1}^H \mathbf{v}_{\mathrm{ss},1}}{|\mathbf{c}_{i,1}^H \mathbf{v}_{\mathrm{ss},1}|}$.

Generalization to multistream requires to solve a so called orthogonal Procrustes problem [12]:

$$\mathbf{R} = \arg\min_{\Omega \in \mathcal{U}_{n_s}} \|\mathbf{V}_{\mathrm{ss},1} - \mathbf{W}_1 \Omega\|_F. \tag{17}$$

The solution is the polar decomposition of $\mathbf{M} = \mathbf{W}_1^H \mathbf{V}_{\mathrm{ss},1}$, which is given by $\mathbf{R} = \tilde{\mathbf{U}}\tilde{\mathbf{V}}^H$ where $\mathbf{M} = \tilde{\mathbf{U}}\Sigma\tilde{\mathbf{V}}^H$ is the singular value decomposition of $\mathbf{M}$.

VI. Simulation results

We numerically evalutate the Shannon capacity of the different schemes when $\mathbf{H}_{\mathrm{ss}}$ is flat with i.i.d Rayleigh fading components. The BSs construct the precoding matrix $\mathbf{W}_{\mathrm{ss}}$ from the feedback bits sent by the MS. As in [5], it is assumed that the BSs knows the large scale path gains of the channels contains in $\mathbf{G}$, and compute the final precoding matrix as $\mathbf{W}_{\mathrm{ls}} = \frac{\sqrt{n_t n_{bs}}}{\|\mathbf{G}\|}\mathbf{G}\mathbf{W}_{\mathrm{ss}}$. With ρ the SNR per-stream, the spectral efficiency is then given by

$$C = \mathcal{E}\left[\log_2 \det\left(\mathbf{I} + \rho \mathbf{W}_{\mathrm{ls}}^H \mathbf{H}_{\mathrm{ls}}^H \mathbf{H}_{\mathrm{ls}} \mathbf{W}_{\mathrm{ls}}\right)\right]. \tag{18}$$

Fig. 2, 3 and 4 depict the spectral efficiency of the proposed scheme for 2 BSs with 2-bit feedback, 2 BSs with 3-bit feedback, and 3 BSs with 2-bit feedback, respectively. We consider equal large scale path loss for each channel. This corresponds to the scenario where the MS is at the cell edge. Cell edge users are more inclined to be served by cooperative transmission, while in this context the performance gap between the different methods is more consequent.

Fig. 5 depicts the variation of performance for different large scale path gain imbalance between the first and the second BS. The simulation scenario is a 2-bit feedback. The graph represents the different performance depending of the position of the user, from the center of the cell to the cell edge.

The optimum Grassmannian codebooks are taken from [10]. Using brute-force search, joint Grassmannian-Stiefel codebooks were generated by maximizing the average Stiefel distance between the representatives of the Grassmannian codebook (maximizing the minimum Stiefel distance was also considered leading to similar but slightly worse spectral efficiency).

As it can be seen from Fig. 2, 3, 4 and 5, the Stiefel-improved codebooks lead to better performances for all codeword selection methods. The proposed independent codeword selection with Stiefel distance consequently improves the scheme and leads to competitive performance w.r.t. the joint selection. Fig. 5 shows that the independent selection methods gain performance relatively to the joint selection when introducing large scale path gain imbalance. Cooperation are more likely to happen when the user is close to the cell edge, which corresponds to the right half of Fig. 5. However for illustration purposes, when the user is at the center of the first cell $\alpha_2/\alpha_1 \approx 0$, the independent selection outperform the joint selection agreeing with the results of [6]. Also, at this point, all the independent selection methods merge which is justified since the signal of the second BS vanishes. Joint selection with transformed codebook provide the best performance in every scenario: matching the performance of independent selection for large imbalance and joint selection for no imbalance.

VII. Conclusion

We have considered product codebook quantization for cooperative base stations. In this context, to improve performance, we have described a joint Grassmannian-Stiefel codebook design. Additionally, to decrease the complexity

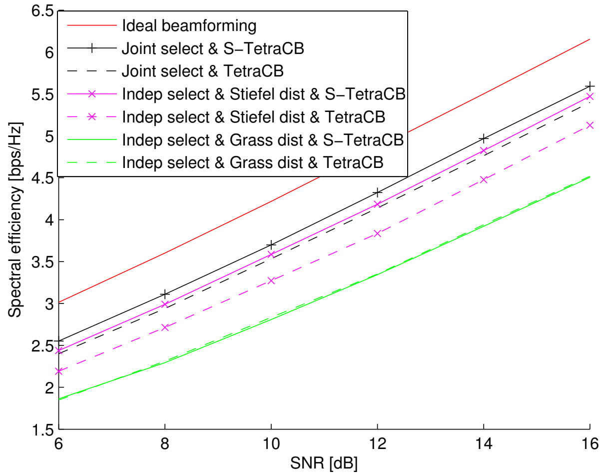

Fig. 2. Performance comparison for 2 BS with 2 Tx antennas using 2-bit Tetrahedron codebook and Stiefel-Tetrahedron codebook

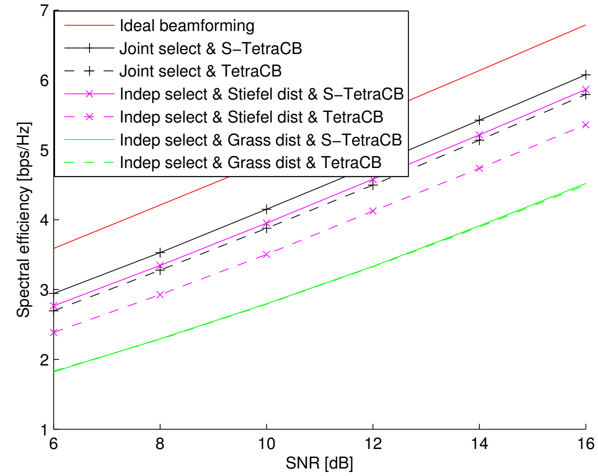

Fig. 4. Performance comparison for 3 BS with 2 Tx antennas using 2-bit Tetrahedron codebook and Stiefel-Tetrahedron codebook

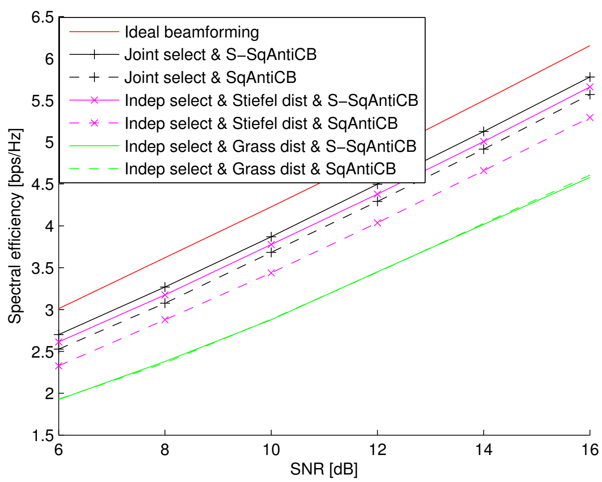

Fig. 3. Performance comparison for 2 BS with 2 Tx antennas using 3-bit Square-Antiprism codebook and Stiefel-Square-Antiprism codebook

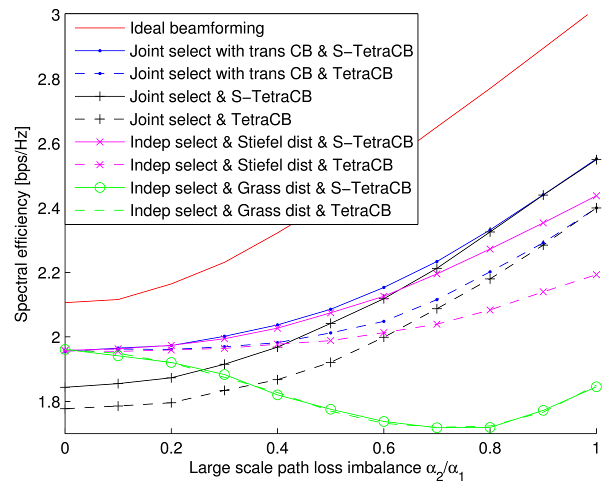

Fig. 5. Performance comparison for 2 BS with 2 Tx antennas using 2-bit Tetrahedron codebook and Stiefel-Tetrahedron codebook as a function of large scale path gains imbalance. The strongest channel is fixed at a SNR of 6 dB

of the codeword selection at the receiver, we considered independent codeword selections, and proposed a selection based on a distance on the Stiefel manifold to absorb in the selection the phase ambiguity between the channels. The pertinence of the scheme was illustrated by simulations.

References

[1] J. H. Kim, W. Zirwas, and M. Haardt, "Efficient feedback via subspace-based channel quantization for distributed cooperative antenna systems with temporally correlated channels," *EURASIP Journal on Advances in Signal Processing*, vol. 2008, no. 847296, p. 13, 2008.

[2] W. Dai, Y. Liu, B. Rider, and V. Lau, "On the information rate of MIMO systems with finite rate channel state feedback using beamforming and power on/off strategy," *IEEE Trans. Inf.*, vol. 55, no. 11, pp. 5032 –5047, nov. 2009.

[3] D. J. Love, R. W. Heath Jr., and T. Strohmer, "Grassmannian beamforming for multiple-input multiple-output wireless systems," *IEEE Trans. Inf.*, vol. 49, no. 10, pp. 2735–2747, Oct. 2003.

[4] L. Thiele, M. Schellmann, T. Wirth, and V. Jungnickel, "Cooperative multi-user MIMO based on limited feedback in downlink ofdm systems," in *Asilomar Conf. on Signals, Systems and Computers*, oct. 2008, pp. 2063 –2067.

[5] Y. Cheng, V. Lau, and Y. Long, "A scalable limited feedback design for network MIMO using per-cell product codebook," *IEEE Trans. Wireless Commun.*, vol. 9, no. 10, pp. 3093 –3099, october 2010.

[6] D. Su, X. Hou, and C. Yang, "Quantization based on per-cell codebook in cooperative multi-cell systems," in *IEEE WCNC*, march 2011, pp. 1753 –1758.

[7] F. Yuan and C. Yang, "Phase ambiguity quantization for per-cell codebook based limited feedback coordinated multi-point transmission systems," in *IEEE VTC Spring*, may 2011, pp. 1 –5.

[8] X. Hou and C. Yang, "Codebook design and selection for multi-cell cooperative transmission limited feedback systems," in *IEEE VTC Spring*, may 2011, pp. 1 –5.

[9] J. H. Conway, R. H. Hardin, and N. J. A. Sloane, "Packing lines, planes, etc.: Packings in Grassmannian space," *Exp. Math*, vol. 5, pp. 139–159, 1996.

[10] R.-A. Pitaval, H.-L. Maattanen, K. Schober, O. Tirkkonen, and R. Wichman, "Beamforming codebooks for two transmit antenna systems based on optimum grassmannian packings," *IEEE Trans. Inf.*, vol. 57, no. 10, pp. 6591 –6602, oct. 2011.

[11] D. Love and J. Heath, R.W., "Limited feedback diversity techniques for correlated channels," *IEEE Trans. Veh. Technol.*, vol. 55, no. 2, pp. 718 – 722, march 2006.

[12] N. J. Higham, "Matrix nearness problems and applications," in *Applications of Matrix Theory, Oxford University Press*, 1989, p. 127.