Repeated Spectrum Sharing Games in Multi-Operator Heterogeneous Networks

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***Abstract*—When femto cells of different network operators do not generate harmful interference to each other, they may agree to aggregate their spectral resources and share spectrum. While this approach would enhance the femto cell performance of both operators, it may introduce harmful inter-operator cross-tier interference, for instance, between the femto cells of one operator and the micro or the macro cells of the other. In this paper, we design a scheme in which two operators construct a spectrum pool and coordinate spectrum sharing in the downlink using repeated games between femto cells and micro cells of different network operators. The scheme does not require operator-specific information exchange. It is based on book keeping of asking and receiving *spectrum usage favors*. When the micro cells of an operator are exposed to high inter-operator interference, they ask for spectrum usage favors from the femto cells of the opponent operator. A spectrum usage favor is exchanged if the micro cells of an operator ask for the permission to use some of the resources of the spectrum pool exclusively, and the femto cells of the opponent operator allow it. Micro cells with a high load may take spectrum usage favors from femto cells that have few users to serve. We illustrate that two heterogeneous network operators can exploit varying network traffic and interference profiles in space and time and both achieves benefits from spectrum sharing.**

***Keywords*—Spectrum sharing, Heterogeneous Network (HetNet), repeated games, spectrum usage favors.**

I. INTRODUCTION

More than 80 % of mobile data traffic originates presently from indoor venues [1]. Indoor performance becomes significantly impacted in case the user equipment (UE) is connected to the macro base station (BS), owing to serious attenuation due to the walls, ceilings and other obstacles. In order to improve the performance, 3GPP has introduced an advance network topology based on HetNets within the framework of LTE-A [2]. A HetNet refers to as a network containing a diverse set of low-powered radio access nodes, also known as small cells, deployed in areas with high data traffic volume for the purpose of locally meeting the capacity demand. Such heterogeneous deployment coupled with spectrum sharing among the macro cell and the small cells can improve the network capacity without requiring additional spectral resources and excessive deployment cost.

In state-of-the-art cellular systems, mobile network operators (MNOs) are allocated spectrum on an exclusive basis. Such allocation facilitates inter-operator interference control but may result in low spectrum utilization in space and time. In hotspot areas, the capacity demand is characterized by high variability, i.e., small cells may not utilize their spectral resources all the time. Also, small cells of different MNOs may offer services in well-separated areas, e.g., different indoor premises. In these cases, inter-operator interference is not harmful and small cells may benefit from inter-operator spectrum sharing.

This paper explores spectrum sharing between two MNOs. Each MNO deploys a HetNet where the indoor femto cells share spectrum with the outdoor micro cells. The deployment is laid in a geographical area containing buildings providing concentrated services and adequate coverage for both indoor and outdoor UEs. The femto cells of different MNOs are geographically separated, do not invest in infrastructure sharing, and therefore can benefit from inter-operator spectrum sharing. However, the sharing implicates strong cross-tier interference between the femto cells of one MNO and the micro cells of the other, especially to the micro-connected UEs deployed in the domain of the other MNO's femto cells. This requires coordination of inter-operator spectrum sharing based on the cross-tier inter-operator interference.

In literature, cooperative spectrum sharing has been studied in the context of a single system, e.g., among coexisting radio links [3], between macro cells and femto cells [4], [5] and also between multiple systems, for instance, different MNOs [6]. In [5], macro and femto BSs exchange channel state information (CSI) to implement coordinated beamforming and attain better spectral efficiency while keeping cross-tier interference under control. Although cooperative binding or infrastructure sharing can lead to high gains, it requires information exchange, e.g., interference prices, CSI, which might be inconvenient among competitive MNOs. Nevertheless, many inter-operator spectrum sharing schemes have been designed based on the principle of revealing operator-specific information, for instance, in [6], two MNOs share their CSI and attain mutual benefits from a pairwise exchange of resource blocks.

Spectrum sharing has also been formulated by means of auction-based and contractual mechanisms, e.g., in [7], macro cell bids for the access permission for high interference inflicted UEs in the femto cells. In [8], the contract theory-based model is used in which the monopolist operator determines the qualities and prices for spectrum with an aim of maximizing its own revenue. In a multi-operator context, MNOs may be hesitant in adopting market-driven sharing schemes as they may be reluctant to touch their revenue model. Moreover, we are interested in adaptive spectrum use on a short timescale, related, e.g., to cell load variations and changing interference conditions. It is a non-trivial task to design such efficient mechanisms for a limited area and a limited time, and to couple operator strategies to the income model of operators. Auction-based schemes may also inhibit the emergence of new types of market players, e.g., local operators, as it would be difficult for them to bargain spectrum with major players.

Among competitive MNOs, a non-cooperative game the-

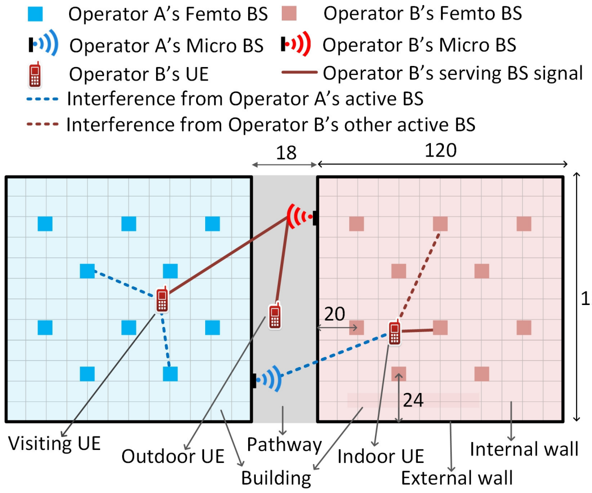

Fig. 1. Inter-operator HetNet deployment scenario.

oretic solution appears to be a more viable option for spectrum sharing. In a non-cooperative framework, MNOs make decisions independently, and the cooperation is entirely self-enforcing. MNOs do not share proprietary information, e.g., network states and optimization targets with other MNOs or external parties and because of that it is not possible to agree in advance about the operational point. However, MNOs must comply to certain spectrum sharing rules either agreed *a priori* between the MNOs or enforced by a legal entity. Also, there is no reason to assume that MNOs are willing to take part in monetary transactions. Under these constraints, we set up repeated games model in which MNOs may take and grant *spectrum usage favors* at each stage game. The term 'favor' captures the notion in which the MNO has performed a favor to someone else, and expects the beneficiary to do the same in future [10]. When spectrum sharing takes place over a spectrum pool, a favor refers to the case where an MNO asks for the permission to use some of the resources of the pool on an exclusive basis. For instance, an MNO with high network load may ask for spectrum usage favors and another MNO that has few UEs to serve may vacate some of the resources of the pool for some time. Unlike spectrum sharing based on one-shot games, in the repeated games model, the action of an MNO at each stage game takes into account not only the immediate rewards but also the history of interactions with the opponent MNO entailing the benefits of reciprocity.

In a previous work [9], the concept of *spectrum usage favor* has been used to coordinate spectrum sharing in the downlink (DL) between femto cells of different MNOs. It is shown that two symmetric MNOs can exploit the varying network load and inter-operator interference conditions and negotiate the utilization of the spectrum pool so that, in the long run, they both achieve benefits as compared to the case with no favors exchanged. In this paper, we show how to extend and apply the spectrum sharing rule based on the book keeping of asking and receiving spectrum usage favors in a multi-operator spectrum sharing HetNets. We design the strategies for the MNOs with a possibility of acquiring favors by the both operators at the same time, unlike in [9], and prove that by adopting these strategies the utility for both MNOs increases in the long run as compared to the case with no exchange of favors.

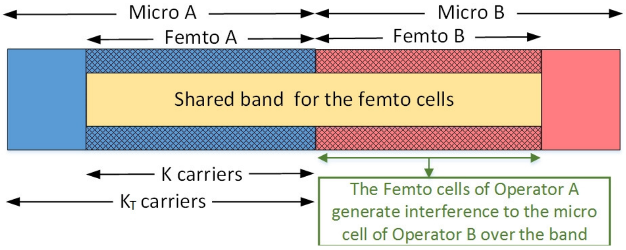

Fig. 2. Inter-operator spectrum sharing scenario.

II. SPECTRUM SHARING SCENARIO AND SYSTEM MODEL

We consider a spectrum sharing scenario between two HetNet operators, Operator A and Operator B, that cover the same geographical area with micro and femto BSs. In some buildings, there may be femto BSs of both operators, in some buildings, only of one operator. Cell selection is based on Reference Signal Received Power. Thus typically the operator's femto network is used to serve indoor UEs in the buildings where there are femto BSs of the operator. The outdoors micro BSs typically located outside the building provide coverage to the outdoor UEs as well as to the indoor UEs served by the operator that are in a building *without* own operator's femto BSs. For simplicity, we concentrate in simulations on a scenario where in each building, there is a femto network of only one operator. It is straight forward to extend to situations with overlapping femto coverage.

Accordingly, there are indoor UEs that are in a building where there is coverage only from the other operator's femto network. We call such UEs *visiting UEs*. An example scenario is depicted in Fig. 1, where in each building there is femto coverage by only one of the operators. Note that the visiting UE of operator B that is in the building with only femto BSs of Operator A is served by the micro BS of Operator B.

Each operator has dedicated licensed spectrum and divides it into K_T component carriers (CCs) of equal bandwidth. The micro BS may transmit in all the K_T CCs whereas, the femto BSs are allowed to transmit in K CCs, $K \leq K_T$. We assume that the femto cells of both operators construct a spectrum pool. Each operator gives the K femto CCs to the pool, so that aggregate bandwidth of $2K$ CCs is available for femto cell usage. This agreement can be attributed to the fact the femto cells have low transmit power levels and with considerable outer building wall attenuation, the inter-operator femto cell interference becomes insignificant. On the other hand, the micro cell does not transmit in the opponent operator's frequency band, i.e., both micro cells share the spectrum orthogonally.

Due to the sufficient separation between the femto cells of different operators, the interference between the femto cells of the operators is decoupled, hence the coordination of the spectrum pool utilization on the femto cell level is not needed [11]. However, due to spectrum sharing among femto cells, the micro-connected UEs may be exposed to harmful inter-operator interference generated by the femto cells of the opponent. This occurs especially for visiting UEs. These UEs are served by the micro network, and suffer serious interference in the K CCs that are part of the pool, where the opponent femto network may operate. The spectrum sharing scenario is depicted in Fig. 2.

When the number of visiting UEs becomes high, the inter-operator interference may become dominant and the overall network performance of the operator may become poor. In that case, the operator will be benefited by using some of the CCs of the pool on an exclusive basis. To counter the harmful inter-operator interference, the micro cell can ask for spectrum usage favors from the femto cells of the opponent. A favor refers to the following action: An operator asks the opponent to stop using for some time some of the K CCs that the operator has contributed to the spectrum pool. In the considered scenario, the micro cells can only ask for favors and the femto cells can only grant favors. Some coarse time synchronization is required so that operators negotiate for spectrum almost simultaneously. An operator can ask and grant favors at the same round of spectrum usage negotiations, i.e., the micro cells of both operators take favors from the opponent operator's femto cells and the utilization of the pool may even become orthogonal.

While negotiating the utilization of spectrum pool, the operators agree on certain rules. First, a favor that is exchanged between the operators necessitates a departure from the default state. Any MAC protocol could be applied in the default state, but for simplicity, we assume that each femto cell transmits over $2K$ CCs with fixed transmit power level per CC. Second, the time period a spectrum usage favor is valid must be fixed and known. We assume that payoff computation and decision making time is small in comparison to the time period of a favor. Moreover, the coherence time of traffic is large enough to expect a radical change in instantaneous loads during the time period of a favor. After this time period expires, the utilization of the pool falls back to the default state and then a new round of negotiations will start. Time scales for a favor would be of the order of the duration of typical UE flows, with re-negotiations on a time scale of seconds. Last, the operators agree about the decision mechanism for asking/granting spectrum favors. It is important to note that operators need not to be aware of each other's network state. The communication rate for indicating the operator's decision to the opponent operator is expected to be negligible.

Given its own network state, Operator $X \in \{A, B\}$ should be able to evaluate how much it gains/loses by taking/granting a favor. To do that, the operator must evaluate its network utility in the default state and also in the hypothetical state after the exchange of 'presumed' favors. To evaluate the latter, the operator must be capable of identifying the effect the opponent operator has on its network utility. For inter-operator spectrum sharing in the DL, this functionality requires that the UEs are able to separate between their own and the other operator's generated interference.

For simplicity, we assume that both operators maintain a proportionally fair utility function u_X directly constructed based on the rates of the UEs served by the operator:

$$u_X = \sum_{n=1}^{n_F} \log\left(\sum_{k=1}^{2K} r_{n,k}\right) + \sum_{n=1}^{n_M} \log\left(\sum_{k=1}^{K_T} r_{n,k}\right) \quad (1)$$

where n_F is the number of UEs served by the femto network and n_M the number of UEs served by the micro network of Operator X. The DL transmission rate of the n-th UE of Operator X on the k-th CC is

$$r_{n,k} = w_{n,k} b_k \log_2 (1 + \gamma_{n,k})$$

where $w_{n,k}$ is the scheduling weight, b_k is the bandwidth of a CC and $\gamma_{n,k}$ is the DL UE SINR defined as

$$\gamma_{n,k} = \frac{c_{X,k}\, S_{n,k}}{I_{n,k} + c_{-X,k}\, I_{-X,n,k}}$$

where $S_{n,k}$ the DL received signal power, $I_{n,k}$ is the power per CC of thermal noise, interference from the operator's own network and external interference and $I_{-X,n,k}$ is the interference level from the opponent operator's interfering BSs. The $c_{X,k}$ is the assignment indicator; $c_{X,k} = 1$ if the CC k is in use by the femto cells of Operator X, and $c_{X,k} = 0$, otherwise. By default, $c_{X,k} = 1$, i.e., both operators use full spectrum pool before negotiating any sharing agreements. Power control is not employed and all the active CCs transmit at the same power level.

Each operator separately performs scheduling to maximize the utility of its UEs. The scheduling is performed jointly for the femto and micro network. For that, the scheduling weights, $w_{n,k}$ are determined as a solution of the optimization problem

$$\begin{aligned} \underset{w_{n,k}}{\text{Maximize}}: & \quad u_X \\ \text{Subject to}: & \quad \sum_{n=1}^{n_F+n_M} w_{n,k} = 1\, \forall k \\ & \quad w_{n,k} \geq 0,\ \forall \{n, k\} \end{aligned}$$

where the two constraints reflect the situation that femto cells and micro cells transmit all the time when they have a UE to serve. The scheduling decision of a BS depends on the quality of the different CCs. Thus a micro BS would not tend to schedule a visiting UE to a CC which is used by the opponent femto network.

III. SPECTRUM SHARING PROTOCOL

The data traffic variations in small cell deployments are expected to be high. In the considered spectrum sharing scenario, see Fig. 1 and Fig. 2, when the number of visiting UEs for Operator B becomes high, and at the same time, the number of indoor femto cell UEs for Operator A is low, Operator A may be willing to vacate some of the K CCs contributed by Operator B to the spectrum pool and curtail excessive inflicted interference on Operator B's visiting UEs. Since we do not consider monetary transactions, Operator A has an incentive to do that only if Operator B is cooperative in return. In the considered scenario, the operators can be cooperative at the same round of negotiations, e.g., they both vacate some CCs from the pool. Furthermore, due to the changing network traffic profiles and the perceived interference, an operator can take favors at some time instant, and return these favors at some point later in future.

During a single stage of negotiations, Operator X may choose an action from the following set: (i) to ask for a favor on $k = 1, \ldots, K$ CCs denoted by a_k, (ii) to grant a favor on $l = 1, \ldots, K$ CCs denoted by g_l if the opponent asks for it, (iii) to simultaneously ask a favor on $k = 1, \ldots, K$ on CCs and grant a favor on $l = 1, \ldots, K$ CCs denoted by $a_k g_l$, or (iv) do neither, denoted by $a_0 g_0$. To specify the outcome of the negotiations, we assume that favors are exchanged only in the following two cases: (a) one player plays a_k and the other plays g_l with $l \geq k$. In that case, k favors are granted to the operator that played a_k. (b) one player plays $a_k g_p$ and the other plays $a_q g_l$ with $l \geq k$ and $p \geq q$. In that case, the operator played $a_k g_p$ takes k favors from the opponent and also grants q favors to the opponent. Obviously, the taken and granted favors are on different CCs.

Based on the outcome of the negotiations the operators draw rewards. (i) the reward when an operator takes a favor is the utility gain of micro cell when the femto-to-micro interference on k CCs diminishes, (ii) the reward when an operator grants a favor is the femto cells utility loss when stopping to use l CCs, (iii) the reward when an operator simultaneously takes and grants a favor is the summation of micro cell utility gain on k CCs and femto cells utility loss on l CCs, and (iv) the reward when a player does not ask nor grant a favor is zero.

If we consider myopic players, each operator always plays (i) and asks for a favor on K CCs but refrains itself to play (ii) in order to maximize its reward. As a result, no exchange of favors occurs and the operators always remain in the default state, i.e., their femto cells occupy $2K$ CCs all the time. However, MNOs are expected to share spectrum for a long time and in many different network states, and also have persistent and publicly known identities. The long-term interaction between selfish players could be modeled by non-cooperative repeated games. In a repeated game, the action at a particular stage game does not only depend on the immediate reward but also on the history of previous rewards [12]. The repeated game under consideration is non-cooperative. It is also Bayesian as each operator's reward depends on a random parameter, namely the configuration of UEs at that time. Because of that, it is difficult to analyze it and identify its equilibrium points. Instead, we propose heuristic threshold-based strategies in which MNOs coordinate their actions through Dynamic Spectrum Management [13] and decide whether to ask and/or grant a favor at each stage. However, the implementation does not quantify under the decision-theoretic framework, see, e.g., [14] or coordination games, see, e.g., [15] as neither it focuses on individual choices of an operator rather seeks a settlement of long-term conflicts in a mutual beneficial way nor the operators necessarily always agree with the decision-makings due to their conflicting interests.

Let us assume that at each stage of the game, an operator computes its immediate micro cell utility gain[†] $\Delta u^{\text{gain}}_{X,k}$ and immediate femto cells utility loss[†] $\Delta u^{\text{loss}}_{X,k}$ that would result if it would get or grant favors on $k = 1, \ldots, K$ CCs. The probability distribution function (PDF) of the utility gain when Operator X gets a favor on k CCs is denoted by $f^{\text{gain}}_{X,k}$ and similarly, the PDF of utility loss when granting a favor on k CCs is denoted by $f^{\text{loss}}_{X,k}$. Since the femto cells of both operators transmit over all $2K$ CCs with fixed power level per CC while negotiating for spectrum, these PDFs depend only on the network state of the own operator's network.

We assume that at each stage game, Operator X first checks whether to ask for a favor on K CCs by comparing its immediate micro cell utility gain $\Delta u^{\text{gain}}_{X,K}$ with a threshold $\theta_{X,K}$. If $\Delta u^{\text{gain}}_{X,K} \leq \theta_{X,K}$, the operator then considers whether to ask a favor on $(K-1)$ CCs, and so forth, until for some k CCs, $\Delta u^{\text{gain}}_{X,k} > \theta_{X,k}$, or no k value yields a gain larger than the respective threshold. The operator then asks for a favor on k CCs, if $k \geq 1$. As a result, the probability that Operator X asks for a favor on k CCs is equal to the probability that the utility gain from taking a favor on $j = (k+1), \ldots, K$ CCs is less than the corresponding thresholds $\theta_{X,j}$, and the utility gain from taking a favor on k CCs is higher than the threshold $\theta_{X,k}$. Thus, the probability to ask a favor on k CCs is

$$P^{\text{ask}}_{X,k} = \prod_{j=k+1}^{K} \int_0^{\theta_{X,j}} f^{\text{gain}}_{X,j} dg_j \int_{\theta_{X,k}}^{\infty} f^{\text{gain}}_{X,k} dg_k \tag{2}$$

The distributions of utility gains from taking favors on different number of CCs are dependent. For instance, if there is a high probability to fetch high gains on $(k-1)$ CCs, then certainly there is also a high probability to fetch high gains on k CCs. To simplify the analysis, we assume the probability distributions of gains and losses for different k independent when deriving $P^{\text{ask}}_{X,k}$. The expected utility gain of micro cells of Operator A by taking favors is

$$\widetilde{U}_{A,M} = \sum_{k=1}^{K} P^{\text{grant}}_{B,k} \int_{\theta_{A,k}}^{\infty} g_k f^{\text{gain}}_{A,k} dg_k \prod_{j=k+1}^{K} \int_0^{\theta_{A,j}} f^{\text{gain}}_{A,j} dg_j. \tag{3}$$

One can express the utility gain of Operator B in similar way.

Following the same threshold-based principle, we assume that Operator X grants a favor on k CCs upon asked, if its immediate femto cell utility loss $\Delta u^{\text{loss}}_{X,k}$ is smaller than a threshold $\lambda_{X,k}$. Thus, the probability to grant a favor on k CCs is

$$P^{\text{grant}}_{X,k} = \int_0^{\lambda_{X,k}} f^{\text{loss}}_{X,k} dl_k. \tag{4}$$

The expected femto cell utility loss of Operator A from granting favors is

$$\widetilde{U}_{A,F} = -\sum_{k=1}^{K} P^{\text{ask}}_{B,k} \int_0^{\lambda_{A,k}} l_k f^{\text{loss}}_{A,k} dl_k. \tag{5}$$

We assume that the networks of the operators are similar, and in symmetric relationship with each other. The operators are patient and do not discount their payoffs, i.e., the discount factor is sufficiently close to 1. To get preliminary understanding on steady state behavior in such a setting, inspired by [16], we thus assume that averaged over long times, operators give and take the same amount of equally valuable favors. Thus, favors would become a rudimentary radio access network-level spectrum sharing currency. We have a steady state equation

$$\sum_{k=1}^{K} k\, P^{\text{ask}}_{A,k} P^{\text{grant}}_{B,k} = \sum_{k=1}^{K} k\, P^{\text{ask}}_{B,k} P^{\text{grant}}_{A,k} \tag{6}$$

where the left- and right-hand side describes the average number of CCs Operator A and Operator B gets a favor on.

In this setting, it is plausible that self-interested operators with a sufficiently long patience, playing the spectrum sharing

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[†] Refer to Appendix VI-A for the calculation of immediate micro cell utility gain $\Delta u^{\text{gain}}_{X,k}$ and immediate femto cells utility loss $\Delta u^{\text{loss}}_{X,k}$.

game repeatedly, would develop strategies of positive reciprocity. Such approaches are known to arise between economic actors [17], corroborating the steady state constraint (6). This constraint might not be well-founded if operators are asymmetric, e.g., they have different utilities, or different network topologies or loads.

To sum up, an operator can progressively estimate the PDFs of utility gain, $f_{X,k}^{\text{gain}}$, and utility loss, $f_{X,k}^{\text{loss}}$, by sampling its own network state and using inter-operator interference measurements. In addition, an operator can keep track of the opponent operator's actions during all stage games and refine its estimates for the probabilities of the opponent for asking and granting spectrum favors, $P_{-X,k}^{\text{ask}}, P_{-X,k}^{\text{grant}}$. In the view of machine learning algorithms, an operator takes an unsupervised learning approach to build models that describe its own network state as well as the behavior of the opponent. These models are used as inputs by the operator while deducing its decision thresholds, $\theta_{X,k}$ and $\lambda_{X,k}$ for satisfying the constraint (6). The thresholds maximizing an excess expected utility $\widetilde{U}_X$ calculated over the Nash Equilibrium (NE) of a one-shot game is chosen. In the NE of the one-shot game, both operators utilize all the $2K$ CCs. The excess utility for an operator reflects its expected gain from taking favors penalized by its expected loss from granting favors. From equations (3) and (5), the excess utility for Operator X can be computed as

$$\widetilde{U}_X = \widetilde{U}_{X,M} + \widetilde{U}_{X,F}, \tag{7}$$

and the optimization problem for identifying the decision thresholds is

$$\begin{aligned} &\underset{\theta_{X,k},\lambda_{X,k}\ \forall k}{\text{Maximize}:} && \widetilde{U}_X \\ &\text{Subject to}: && \text{Eq. (6).} \end{aligned} \tag{8}$$

In order to solve the optimization problem (8), we construct the Lagrangian function and evaluate the first-order conditions. Also, we compute the Lagrangian at the borders, and finally select the point that maximizes the Lagrangian. The Lagrangian optimization has low complexity because the decision thresholds $\theta_{X,k}, \lambda_{X,k}$ can be computed in closed-form as functions of the Lagrange multiplier. The complete solution to the given optimization problem is given in Appendix VI-B. In the Appendix, it is also shown that operators following the threshold-based strategy with thresholds calculated based on the first-order conditions achieve benefits as compared to the case with no exchange of favors.

IV. NUMERICAL EXAMPLES

We study the UE rate improvement due to spectrum sharing for two HetNet operators employing the spectrum sharing protocol described in Section III. The simulation scenario as depicted in Fig. 1, each building covers an area of 120 × 120 m^2 which is divided into a 12 × 12 grid of identical rooms, each with an area of 10 × 10 m^2. Rooms are partitioned by walls introducing 5 dB attenuation while outer-building walls introduce attenuation equal to 15 dB [18]. Each building houses 10 femto BSs belonging to a single operator and providing services to the indoor UEs. The same operator's micro BS is placed outside of the building in order to provide adequate coverage to the outdoor UEs in the pathway and also to the visiting UEs in the opponent operator's building. The antenna pattern for the femto BSs is omnidirectional whereas micro BSs have 180° sector antennas.

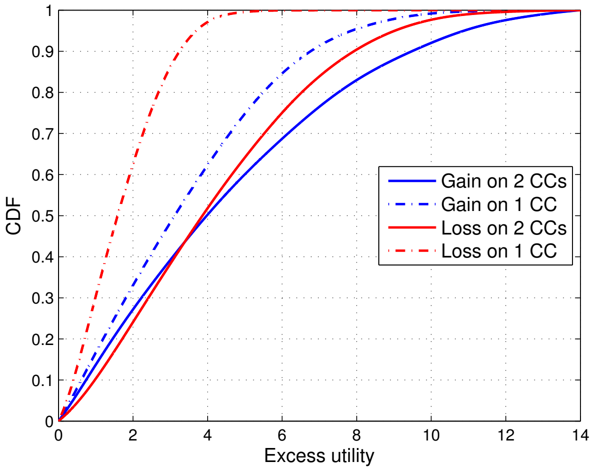

Fig. 3. Distribution of utility gains and utility losses for Operator A during the progression of the game.

We consider propagation at 2.6 GHz based on a power-law model for distance-based propagation pathloss with attenuation constant 10^{-4} and pathloss exponent 3.7. Each operator has a dedicated license to use a bandwidth of 30 MHz in the DL which is split into three equally-sized CCs of $b_k = 10$ MHz. The micro BS of an operator can transmit over all three CCs, i.e., $K_T = 3$ whereas the femto BSs can transmit only over two CCs, i.e., $K = 2$. The available power budget per 10 MHz for micro BS is 30 dBm whereas for femto BS is 20 dBm. The thermal noise power level in a 10 MHz bandwidth is -93 dBm.

Each operator contributes two CCs used by the femto BSs giving rise to a pool of 40 MHz. The femto BSs would tend to use the full spectrum pool owing to non-harmful interference between femto cells of different operators. However, the micro BS may face intense interference from the opponent operator's femto BSs over the half of the pool's spectral resources, i.e., on the 20 MHz band, see also Fig. 2. In that case, according to the spectrum sharing protocol discussed in Section III, the operators ask for spectrum usage favors.

Initially, we generate the distributions of utility gains and losses over 200 000 simulation snapshots or equivalently 200 000 stage games. At each stage game, the operators calculate and keep track of their utility gains and losses for the assumed favors on one and two CCs in order to progressively construct the distributions of utility gains and losses, shown in Fig. 3. Over these snapshots, we simulate many different network realizations including symmetric and asymmetric network loads between the operators, cases with high and low visiting probability, etc. so that the distributions of utility gains and utility losses can be seen as the steady state distributions. With the steady state distributions at hand, we can evaluate the performance of the spectrum sharing scheme in terms of UE rate cumulative distribution function (CDF). We consider a finite time horizon of 2000 games. During these games the network traffic and interference profiles can also vary.

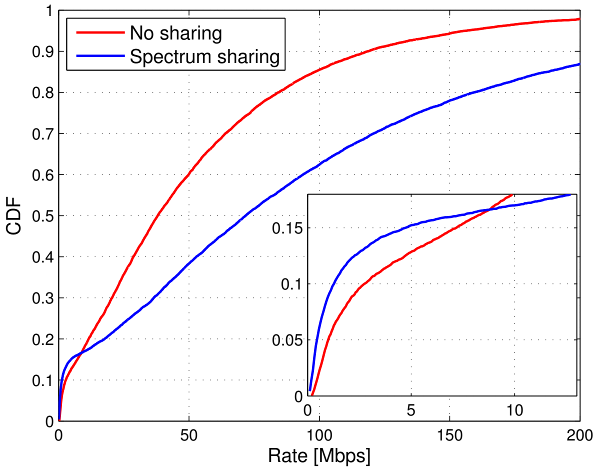

Fig. 4. Rate distribution for the UEs of an operator. Low visiting probability for the operators.

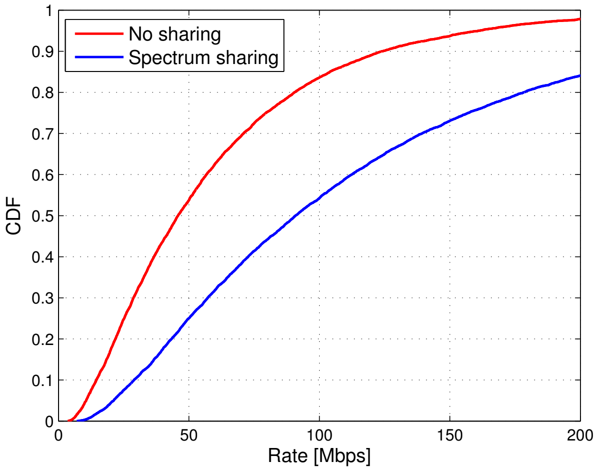

Fig. 5. Rate distribution for the UEs of an operator. Zero visiting probability for the operators.

At the beginning, the decision thresholds for asking and granting favors are set arbitrarily equal to $\theta_{X,1} = \theta_{X,2} = 1$ and $\lambda_{X,1} = \lambda_{X,2} = 1$ for both operators. The operators recompute the opponent operator's probabilities for asking and granting favors considering all previous stage games, for instance, in the simulation setup, the asking probability for a favor on k CCs is computed as 'the number of times an opponent operator asks the favor/total number of executed stage games', whereas the granting probability for the same is 'the number of times an opponent operator grants the favor upon asking/the number of times an operator asks the favor'. Then, the decision thresholds are updated by solving the optimization problem (8) and decisions are made whether to ask a favor or not, and/or to grant a favor or not upon asking. After the decisions, carrier allocations are updated at each stage game; the operators compute and keep track of the UE rates. Recall from Section II that granted favors are valid only for a particular stage game. At the end of each stage, the CC allocation returns to the default state, i.e., the femto cells utilize all the four CCs of the pool. The performance of the spectrum sharing scheme is assessed in comparison with the conventional scheme where the operators do not participate in spectrum sharing.

First, we consider a scenario with a low visiting probability for both operators, i.e., the network load of the operators in their respective building is considerably higher than in their opponent operator's building. In each stage game, the number of UEs for each operator is drawn from a Poisson distribution with mean eight for indoor femto UEs in the building served by the operator's femto network, mean one for outdoor UEs and mean two for indoor visiting UEs in the building with opponent femto network. The UEs are uniformly distributed in their respective areas. In Fig. 4, the rate distribution curves for the UEs of an operator are depicted. One can see that the femto cell UEs attain significantly better performance as compared to no sharing, at the cost of small degradation in the performance of micro-connected visiting, shown with a zoomed-in view in Fig. 4. On average, 18 % of the UEs of an operator are visiting UEs, and in the lower-tail of the distribution one can see that they face a small reduction in their rates due to the received interference from the femto cells of the opponent. Hence with few or no visiting UEs, no favors are exchanged and the femto cells continue to enjoy high rates. Overall, the femto cells use 100 % more spectrum as compared to no sharing and their mean rate improves by 80 %. Fig. 5 depicts the rate distribution curves for the UEs of an operator with zero visiting probability.

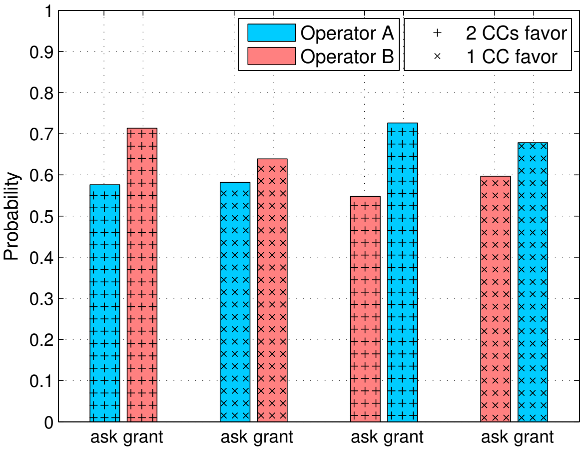

Fig. 6. Probabilities of asking and granting favors in the extreme scenario with population inversion between the operators.

Next, we consider an extreme scenario with a high visiting probability for both operators where most of the UEs in the femto-network area covered by Operator A would be UEs of Operator B and vice versa. The mean number of indoor femto UEs, outdoor UEs and indoor visiting UEs in the building with opponent femto network for each operator are two, one and eight respectively. On average, 72 % of the UEs of an operator are located in the building of the opponent operator. Because of the strong interfering networks, operators should mostly agree to orthogonalize the spectrum pool. Fig. 6 illustrates that operators ask favors on one or two CCs with a probability

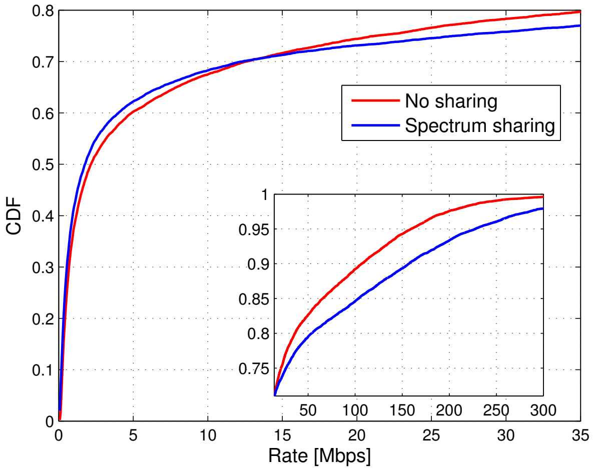

Fig. 7. Rate distribution for the UEs of an operator. Extreme scenario depicting high visiting probability amongst the operators.

ranging from 55 % to 60 % whereas they grant favors with a probability 64 % to 71 %. Resultant, in Fig. 7, one can see that micro-connected UEs (up-till the intersection at 0.7 of the CDFs) experience diminished losses of 15 % at the $1/2$ of the CDF owing to the fact that the operator receives large number of favors on multiple CCs due to the strong micro cell load. Without exchange of favors, the micro cell UEs bear performance loss of 46 % if the both operators use full spectrum pool. On the other hand, the femto cells utilize 40 % more spectrum as compared to no sharing and they still enjoy a high performance gain close to 45 %, with a zoomed-in view in Fig. 7.

V. CONCLUSION

Femto cells of different operators may be willing to share spectrum when their mutual interference is not harmful, e.g., different operators deploy femto cells in well-separated areas and/or femto cells utilize higher frequencies e.g., millimeter waves. Nevertheless, the femto cells of an operator can still generate harmful interference to the users of another operator that are served by a different layer, e.g., outdoor urban micro cells providing access to users located in the hotspot area of another operator. In that case, the femto cells and the micro cells of different operators need to coordinate spectrum sharing between each other. Ideally, this coordination should not touch the revenue model of operators, it should have low complexity and should not reveal operator-specific information among operators and/or to other parties. We designed a repeated game sharing mechanism that fulfills these properties and takes advantage of the varying network traffic and interference profiles in space and time. When the micro cells are not heavily loaded, the femto cells can entertain the benefit from spectrum sharing. On the other hand, when the micro cells become heavily loaded and exposed to high inter-operator interference, the femto cells of the other operator may stop using some of their spectral resources. The incentive to do that is that the operator will also be cooperative in return. Exploiting short- and long-term reciprocity, we illustrated that two heterogeneous network operators with similar deployment densities have incentive to be cooperative.

VI. APPENDIX

A. *Calculation of immediate utility gain and loss*

The utility function u_X in (1) is defined over the CC assignment indicator $c_{X,k}$ and $c_{-X,k}$ for $k=1,\ldots,K$ CCs, and thus can be represented as

$$u_X = u_X\left(\boldsymbol{c}_X, \boldsymbol{c}_{-X}\right)$$

where $\boldsymbol{c}_X$ is the assignment vector, $\boldsymbol{c}_X = (c_{X,1},\ldots,c_{X,K})$. In the default state, operators use the full spectrum pool, and therefore all the elements in $\boldsymbol{c}_X$ are one, i.e., $\boldsymbol{c}_X = \mathbf{1}^K$ where $\mathbf{1}^K$ is the row vector with all enteries equal to unity.

The vector $\boldsymbol{c}_X$ can be re-written as $\boldsymbol{c}_X = (\boldsymbol{c}_X^k, \boldsymbol{c}_X^{K-k})$ where $\boldsymbol{c}_X^k = (c_{X,1},\ldots,c_{X,k})$ and $\boldsymbol{c}_X^{K-k} = (c_{X,k+1},\ldots,c_{X,K})$. Operator X estimates its utility gain for a favor on k CCs assuming that the opponent does not transmit on these CCs, i.e., $\boldsymbol{c}_{-X}^k = \mathbf{0}^k$. Thus, the immediate micro cell utility gain for Operator X is

$$\Delta u^{\text{gain}}_{X,k} = u_X\left(\mathbf{1}^K, \left(\mathbf{0}^k, \mathbf{1}^{K-k}\right)\right) - u_X\left(\mathbf{1}^K, \mathbf{1}^K\right). \tag{9}$$

Similarly, Operator X estimates its immediate utility loss if it gives a favor on k CCs, which is

$$\Delta u^{\text{loss}}_{X,k} = u_X\left(\mathbf{1}^K, \mathbf{1}^K\right) - u_X\left(\left(\mathbf{0}^k, \mathbf{1}^{K-k}\right), \mathbf{1}^K\right). \tag{10}$$

In order to construct the utility gain/loss PDFs, operators obtain the instantaneous utility gain (9) and loss (10) for $k = 1,\ldots,K$ CCs. Hence at each stage game, operators collect $2K$ utility gain/loss statistics, and improve the sample size of all $2K$ distributions by one unit to build their distributions with time.

B. *Solution to the optimization problem*

The Lagrangian function of the optimization problem in (8) for Operator A can be formulated as

$$\mathcal{L}_A = \widetilde{U}_A - \mu_A \sum_{k=1}^{K} k \left(P^{\text{ask}}_{A,k} P^{\text{grant}}_{B,k} - P^{\text{ask}}_{B,k} P^{\text{grant}}_{A,k}\right) \tag{11}$$

where μ_A is the Lagrange multiplier.

Using equation (11), and taking the partial derivative with respect to the loss threshold $\lambda_{A,1}$ and setting it equal to zero, allows the computation of the decision threshold $\lambda_{A,1}$, $\lambda_{A,1} = \mu_A$. Setting the partial derivative of the Lagrangian with respect to $\lambda_{A,k} : k > 1$ equal to zero, and substituting the value of the Lagrange multiplier into the resulting equation gives

$$\lambda_{A,k} = k\lambda_{A,1},\ k>1. \tag{12}$$

Next, starting from $\partial\mathcal{L}_A/\partial\theta_{A,1} = 0$, the gain threshold $\theta_{A,1}$ is computed as

$$\theta_{A,1} = \lambda_{A,1}. \tag{13}$$

Finally, setting $\partial \mathcal{L}_A / \partial \theta_{A,k} = 0, k > 1$ and using the solution for $\theta_{A,k-1}$, we compute gain threshold $\theta_{A,k}$ as a function of $\theta_{A,k-1}$, and loss thresholds $\lambda_{A,k}$ and $\lambda_{A,k-1}$

$$\begin{aligned}\theta_{A,k} = \lambda_{A,k} & \\ + \frac{P^{\text{grant}}_{B,k-1}}{P^{\text{grant}}_{B,k}} & \left(\int_{\theta_{A,k-1}}^{\infty} \left(g_{k-1} - \lambda_{A,k-1}\right) f^{\text{gain}}_{A,k-1} dg_{k-1} \right. \\ & \left. + \left(\theta_{A,k-1} - \lambda_{A,k-1}\right) \int_0^{\theta_{A,k-1}} f^{\text{gain}}_{A,k-1} dg_{k-1} \right).\end{aligned} \tag{14}$$

The thresholds $\lambda_{A,k}, \theta_{A,k} \forall k$ that may maximize the Lagrangian must jointly satisfy equations (12)$-$(14) and also the constraint (6). Note that the above system of equations does not accept a closed-form solution but it is straightforward to solve numerically. Besides the calculation of the Lagrangian at the stationary point, we also compute it at the borders. The thresholds, either interior or border, maximizing the Lagrangian are selected.

Next, we show that the obtained solution based on the first-order conditions satisfies $\widetilde{U}_A > 0$, i.e., the operators achieve better performance in comparison to no exchange of favors. To begin with, using integration by parts, equation (5) rewritten as

$$\widetilde{U}_{A,F} = \sum_{k=1}^{K} P^{\text{ask}}_{B,k} \left(\int_0^{\lambda_{A,k}} \mathcal{F}^{\text{loss}}_{A,k} dl_k - \lambda_{A,k} \int_0^{\lambda_{A,k}} f^{\text{loss}}_{A,k} dl_k \right) \tag{15}$$

where $\mathcal{F}^{\text{loss}}_{A,k} = \int f^{\text{loss}}_{A,k} dl_k$.

According to the definition of the probabilities of granting a favor from equation (4), we note that the term acceding the minus sign in equation (15) times $P^{\text{ask}}_{B,k}$ is equal to the right-hand side of the constraint in equation (6) scaled by $\lambda_{A,1}$ and replacing the same, we end up with

$$\begin{aligned}\widetilde{U}_{A,F} \;=\; \sum_{k=1}^{K} P^{\text{grant}}_{B,k} & \left(\int_0^{\lambda_{A,k}} \mathcal{F}^{\text{loss}}_{A,k} dl_k \right. \\ & \left. - \lambda_{A,k} \int_{\theta_{A,k}}^{\infty} f^{\text{gain}}_{A,k} dg_k \prod_{j=k+1}^{K} \int_0^{\theta_{A,j}} f^{\text{gain}}_{A,j} dg_j \right).\end{aligned} \tag{16}$$

Using equation (3) and (16) into equation (7), the excess utility can be read as

$$\begin{aligned}\widetilde{U}_A = \sum_{k=1}^{K} P^{\text{grant}}_{B,k} & \left(\prod_{j=k+1}^{K} \int_0^{\theta_{A,j}} f^{\text{gain}}_{A,j} dg_j \int_{\theta_{A,k}}^{\infty} \left(g_k - \lambda_{A,k}\right) f^{\text{gain}}_{A,k} dg_k \right. \\ & \left. + \int_0^{\lambda_{A,k}} \mathcal{F}^{\text{loss}}_{A,k} dl_k \right)\end{aligned}$$

which is always positive since $\theta_{A,k} \geq \lambda_{A,k} \; \forall k$.

It is a matter of future study to show that the obtained solution is a NE of the infinitely repeated games.

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