Multiple Primary User Spectrum Sensing for Unknown Noise Statistics

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***Abstract*—Multi-antenna spectrum sensing algorithms for cognitive radio are receiving a lot of attention recently. In this paper, we consider multi-antenna detection when the noise covariance matrix is assumed to be arbitrary and unknown. The studies leading to this paper have been motivated by the existence but typically unknown noise correlation in practice. A multiple primary user detector, derived from the generalized likelihood ratio criterion, is analyzed in such a scenario. We calculate the exact moments of the test statistics involved, which lead to a simple and accurate analytical formula for the false alarm probability. The result is obtained by utilizing tools from multivariate analysis as well as moment based approximations. Simulations are conducted to examine accuracy of the derived result, with the achieved accuracy being reasonably good. From the considered simulation settings, performance gain over existing detection algorithms is observed in scenarios with arbitrary but unknown noise correlation and multiple primary users.**

***Index Terms*—Cognitive radio; multiple primary users; robust statistics; spectrum sensing.**

I. Introduction

In Cognitive Radio (CR) networks, dynamic spectrum access is implemented to mitigate spectrum scarcity. A secondary (unlicensed) user is allowed to utilize the spectrum resources when it does not cause intolerable interference to the primary (licensed) users. Spectrum sensing is the first key step towards this dynamic spectrum access scenario. Various sensing algorithms can be derived under different modeling assumptions. These assumptions, though, may not often be met in realistic scenarios. Consequently, algorithms that are robust to deviations from the presumed assumptions are of particular interest.

Prior work on multi-antenna cooperative spectrum sensing predominately employ the assumption of known noise covariance matrix. Based on this assumption, several eigenvalue based spectrum sensing algorithms have been proposed recently [1–5]. The assumption of a perfectly known noise covariance matrix may not be realistic for practical systems due to the time-varying nature of the noise statistics. The time-varying noise can be induced, for example, from the unpredictable interferences. Using the existing detection algorithms [1–5] in such a scenario will induce performance loss. In this paper the noise covariance matrix is assumed to be arbitrary but unknown. The true noise covariance matrix is estimated by periodically updated noise-only observations. Detectors derived under this setting, a.k.a. the blind-noise-statistics detectors, are robust to modeling assumptions of the noise covariance matrix. Despite the practical importance of the blind-noise-statistics detectors, results in this direction are rather limited [6, 8, 9]. A heuristic detector based on Roy's statistics [7] was considered in [8, 9]. However, this detector is only useful when there is one active primary user. The assumption of a single primary user may fail to reflect the situations in forthcoming CR networks, where the primary system could be a cellular network, and the existence of more than one primary users would be the prevailing condition.

To address this challenge, we consider a detector for arbitrary and unknown noise covariance matrix in the presence of multiple primary users. The considered blind-noise-statistics detector was proposed in literature under the Generalized Likelihood Ratio (GLR) criterion. We investigate its detection performance by deriving a closed-form approximation to the false alarm probability. The derived result is easily computable and reasonably accurate. Simulations show the robustness of the proposed detector for arbitrary noise covariance matrix and in the presence of multiple primary users.

II. Signal Model

Consider the standard model for K-sensor cooperative detection[1] in the presence of P primary users,

$$\mathbf{x} = \mathbf{H}\mathbf{s} + \mathbf{n}, \tag{1}$$

where $\mathbf{x} \in \mathbb{C}^K$ is the received data vector. The $K \times 1$ vector $\mathbf{n}$ is the complex Gaussian noise with zero mean and covariance matrix $\mathbf{\Psi}$. The $K \times P$ matrix $\mathbf{H} = [\mathbf{h}_1, \ldots, \mathbf{h}_P]$ represents the channels between the P primary users and the K sensors. The $P \times 1$ vector $\mathbf{s} = [s_1, \ldots, s_P]'$ denotes the transmitted signals from the primary users, which is assumed to follow an i.i.d zero mean Gaussian distribution and is uncorrelated with the noise [1–5]. This assumption, for instance, is nearly valid for an OFDM signal in which each carrier is modulated by independent data streams. We further assume that the channel $\mathbf{H}$ is constant during sensing i.e. deterministic channels.

[1]This collaborative sensing scenario is more relevant when the K sensors are in one device. For distributed collaborating sensors, accurate time synchronization between devices and communications to the fusion center become an issue.

We collect N independent observations from model (1) to a $K \times N$ ($K \leq N$) received data matrix $\mathbf{X} = [\mathbf{x}_1, \ldots, \mathbf{x}_N]$. By the above assumptions, the sample covariance matrix[2] $\mathbf{R} = \mathbf{X}\mathbf{X}^\dagger$ of the received data matrix follows a complex Wishart distribution, denoted by $\mathbf{R} \sim \mathcal{W}_K(N, \mathbf{\Sigma})$, with the corresponding population covariance matrix calculated in the absence of primary users, denoted by hypothesis $\mathcal{H}_0$, as

$$\mathcal{H}_0: \qquad \mathbf{\Sigma} := \mathbb{E}[\mathbf{X}\mathbf{X}^\dagger]/N = \mathbf{\Psi}, \tag{2}$$

and in the presence of primary users, denoted by hypothesis $\mathcal{H}_1$, as

$$\mathcal{H}_1: \qquad \mathbf{\Sigma} = \mathbf{\Psi} + \sum_{i=1}^{P} \gamma_i \mathbf{h}_i \mathbf{h}_i^\dagger. \tag{3}$$

Here $\gamma_i := \mathbb{E}[s_i s_i^\dagger]$ defines the transmission power of the i-th primary user and the received SNR of primary user i across the K sensors is defined as $\text{SNR}_i := \frac{\gamma_i ||\mathbf{h}_i||^2}{\text{tr}(\mathbf{\Psi})/K}$, where $\text{tr}(\cdot)$ is the matrix trace operation. Finally, we note that declaring wrongly $\mathcal{H}_0$, or declaring correctly $\mathcal{H}_1$, defines the false alarm probability P_{fa}, and the detection probability P_{d}, respectively.

III. Test Statistics

The differences between the population covariance matrices under $\mathcal{H}_0$ (2) and under $\mathcal{H}_1$ (3) can be explored to detect the primary users.

In the case of a known noise covariance matrix $\mathbf{\Psi}$, without loss of generality [10, pp. 338], we assume that the noise of each sensor is independent of each other and with a common noise power σ^2 i.e. $\mathbf{\Psi} = \sigma^2 \mathbf{I}_K$. In this case, the sufficient statistics is the sample covariance matrix $\mathbf{R}$ of the received data matrix [10] and we denote its ordered eigenvalues by $0 \leq \lambda_K \leq \ldots \leq \lambda_1 < \infty$. In the presence of *a single* primary user, $P = 1$, the Largest Eigenvalue based (LE) detector $T_{\text{LE}} := \lambda_1$ and the Scaled Largest Eigenvalue based (SLE) detection $T_{\text{SLE}} := \lambda_1 / \sum_{i=1}^{K} \lambda_i$, were proposed under the GLR criterion when the noise power σ^2 is assumed to be known [1] and unknown [2], respectively.[3] Detection without assuming any knowledge of certain parameter is often referred to as blind detection. For example the SLE detector belongs to the blind σ^2 detection, which is more robust than the LE detector to the noise power uncertainty. In the presence of multiple primary users, when $P \geq 2$ but not known a priori, i.e. blind P detection, the corresponding detector derived from the GLR criterion is the Spherical Test based (ST) detector[4] $T_{\text{ST}} := |\mathbf{R}| / \left(\frac{1}{K}\text{tr}(\mathbf{R})\right)^K = \prod_{i=1}^{K} \lambda_i / \left(\frac{1}{K}\sum_{i=1}^{K} \lambda_i\right)^K$, which is also a blind σ^2 detector [4]. Performance of the ST detector has been analytically addressed in [5]. Note that for arbitrary but known P, the corresponding GLR detectors have been derived in [11].

The blindness of detection is extended in a new dimension by assuming an arbitrary but unknown noise covariance matrix. The resulting detectors belong to the so-called blind $\mathbf{\Psi}$ detection[5], which are robust to any modeling assumptions on $\mathbf{\Psi}$. This extension is partially motivated by the existence but usually unknown noise correlation due to e.g. antenna coupling in practical systems. Instead of a perfectly known $\mathbf{\Psi}$, we assume to have, in addition to the received data matrix $\mathbf{X}$, another independent noise-only observation matrix $\mathbf{Z}$ consisting of M samples from the K sensors. This noise-only observation matrix $\mathbf{Z}$ can be obtained e.g. when absence of the primary users is declared from an initial coarse sensing period. Moreover, when the signals of interest are narrow-band and located in a known frequency band, such as the case of TV primary systems, the only-noise samples collected at a frequency just outside this band can be justified as having the same noise covariance characteristic. The time-varying nature of the noise correlation can be coped with periodically updating the measurement $\mathbf{Z}$. The unknown noise population covariance matrix $\mathbf{\Psi}$ can be estimated via the noise-only sample covariance matrix $\mathbf{E} := \mathbf{Z}\mathbf{Z}^\dagger$, which, by the assumptions in Section II, follows a complex Wishart distribution i.e. $\mathbf{E} \sim \mathcal{W}_K(M, \mathbf{\Psi})$. In this setting, the sufficient statistics is the 'whitened' sample covariance matrix of the form $\mathbf{E}^{-1}\mathbf{R}$ [10] and its ordered eigenvalues are denoted by $0 \leq \theta_K \leq \ldots \leq \theta_1 < \infty$.

In the scenario of *a single* primary user, a reasonable test statistics to choose is Roy's largest eigenvalue based detector [7] $T_{\text{R}} := \theta_1$. Nadler et al. [8, 9] were among the first to consider Roy's detector in the spectrum sensing application, and derived novel analytical expressions for the detection probability. Although Roy's detector belongs to the blind $\mathbf{\Psi}$ detection, it is not a blind P detector. It is of interest to extend the blindness of the detection to the practical scenario of multiple primary users. It turns out that, under the assumptions of arbitrary but unknown $\mathbf{\Psi}$ and multiple primary users, the corresponding detector constructed from the GLR criterion is Wilks' detector [12]

$$T_{\text{W}} := \frac{|\mathbf{E}|}{|\mathbf{R} + \mathbf{E}|} = \prod_{i=1}^{K} \frac{1}{1 + \theta_i}, \tag{4}$$

where it can be verified that $T_{\text{W}} \in [0, 1]$. The resulting test procedure is

$$T_{\text{W}} \underset{\mathcal{H}_1}{\overset{\mathcal{H}_0}{\gtrless}} \zeta, \tag{5}$$

where ζ is a threshold. The T_{W} detector is blind to both $\mathbf{\Psi}$ and P i.e. its detection performance is robust to the degree of noise correlation as well as to the number of primary users, which renders it the most robust detector under the framework developed in this paper.

IV. Performance Analysis

In this section we propose closed-form approximations to the false alarm probability and the decision threshold of Wilks'

[2] $(\cdot)^\dagger$ denotes the conjugate-transpose operation.

[3] Note that in [1] the problem of unknown noise variance σ^2 was considered as well.

[4] $|\cdot|$ defines the matrix determinant operation.

[5] Strictly speaking, the concept of blindness here is different from those of blind σ^2 or blind P, since the knowledge of $\mathbf{\Psi}$ has been utilized in the form of noise-only samples. In the context of this paper, blind $\mathbf{\Psi}$ refers to the fact that no assumption on the structure of $\mathbf{\Psi}$ is made.

detector based on a derived exact moment expression.

A. Exact Moment Expression

Recall the definition of random variable

$$T_{\mathrm{W}} := \frac{|\mathbf{E}|}{|\mathbf{R}+\mathbf{E}|} \in [0,1], \tag{6}$$

where $\mathbf{R} \sim \mathcal{W}_K(N, \mathbf{\Sigma})$, $\mathbf{E} \sim \mathcal{W}_K(M, \mathbf{\Psi})$ and under $\mathcal{H}_0$ it holds that $\mathbf{\Sigma} = \mathbf{\Psi}$. For a Wishart matrix $\mathbf{R} \sim \mathcal{W}_K(N, \mathbf{\Sigma})$, the density function is defined as

$$f(\mathbf{R}) := \frac{|\mathbf{\Sigma}|^{-N}}{\Gamma_K(N)} |\mathbf{R}|^{N-K} \mathrm{e}^{-\mathrm{tr}\left(\mathbf{\Sigma}^{-1}\mathbf{R}\right)}, \tag{7}$$

where $\Gamma_K(N) = \pi^{\frac{1}{2}K(K-1)} \prod_{j=0}^{K-1} \Gamma(N-j)$ and $\Gamma(\cdot)$ defines the Gamma function. Using the definition (7) and the fact that $\mathbf{R}$ and $\mathbf{E}$ are independent, the m-th moment of T_{W} under $\mathcal{H}_0$ can be calculated as shown on top of the next page, where now $\mathbf{E}' \sim \mathcal{W}_K(M+m, \mathbf{\Psi})$. Since the Wishart matrices $\mathbf{R}$ and $\mathbf{E}'$ have the same covariance matrix $\mathbf{\Psi}$ and dimension K, their sum $\mathbf{R}+\mathbf{E}'$ also follows a Wishart distribution [10, Th. 3.2.4], i.e. $\mathbf{R}+\mathbf{E}' \sim \mathcal{W}_K(N+M+m, \mathbf{\Psi})$. Now using the result for moments of determinant of complex Wishart matrices [13] we have

$$\mathbb{E}[|\mathbf{R}+\mathbf{E}'|^{-m}] = \frac{|\mathbf{\Psi}|^{-m} \Gamma_K(N+M)}{\Gamma_K(N+M+m)}. \tag{11}$$

Inserting (11) into (10), the exact m-th moment of random variable T_{W}, denoted by $\mathcal{M}_m$, equals

$$\mathcal{M}_m := \frac{\Gamma_K(N+M)\Gamma_K(M+m)}{\Gamma_K(M)\Gamma_K(N+M+m)}. \tag{12}$$

B. Moment Based Approximation

Despite the fact that the problem of finding the T_{W} distribution under $\mathcal{H}_0$ has received much attention e.g. in [14–16], this problem is far from being completely settled. For example, the exact density representation [14] via the Meijer G-function is, although of theoretical interest, too complicated for computational purpose. Moreover, for complex Wishart matrices, exact T_{W} densities were derived for a few limited cases, i.e. $K = 2$ and $K = 3$ in [15]. Since an exact and computable distribution of T_{W} seems intractable to obtain for arbitrary parameter values, we will construct a simple yet accurate approximative T_{W} distribution by the moment matching techniques [5]. Contrary to the previously discussed results, the proposed closed-form approximation is valid for any K, N and M.

Motivated by the fact that the exact densities for $K = 2$ and $K = 3$ in [15] hold the same polynomial form $x^i(1-x)^j$ as the Beta density, we choose the Beta distribution to approximate the distribution of T_{W} for general parameter values. An additional motivation is due to the fact the Beta random variable has the same support as that of T_{W}. Specifically, for a Beta random variable with density function $f_{\mathrm{B}}(x) := x^{\alpha_0-1}(1-x)^{\beta_0-1}/B(\alpha_0, \beta_0),\ x \in [0,1]$, the m-th moment is given by

$$\mathbb{E}[x^m] := \int_0^1 x^m f_{\mathrm{B}}(x)\ \mathrm{d}x = \frac{(\alpha_0)_m}{(\alpha_0+\beta_0)_m}, \tag{13}$$

where $B(a,b) = \Gamma(a)\Gamma(b)/\Gamma(a+b)$, $(\alpha)_m = \Gamma(\alpha+m)/\Gamma(\alpha)$ define the Beta function and the Pochhammer symbol, respectively. In particular, by matching the first two moments in (13) to those of T_{W} in (12), the parameters α_0 and β_0 are solved as

$$\alpha_0 = \frac{\mathcal{M}_1(\mathcal{M}_1 - \mathcal{M}_2)}{\mathcal{M}_2 - (\mathcal{M}_1)^2}, \quad \beta_0 = \frac{(1-\mathcal{M}_1)(\mathcal{M}_1 - \mathcal{M}_2)}{\mathcal{M}_2 - (\mathcal{M}_1)^2}. \tag{14}$$

As a result, the two-moment-based approximation to the CDF of T_{W} under $\mathcal{H}_0$ is

$$F_{\mathrm{W}}(y) \approx \frac{1}{B(\alpha_0,\beta_0)} \int_0^y x^{\alpha_0-1}(1-x)^{\beta_0-1} \mathrm{d}x \tag{15}$$

$$= \frac{B(y;\alpha_0,\beta_0)}{B(\alpha_0,\beta_0)}, \tag{16}$$

where $y \in [0,1]$ and $B(x;a,b) = \int_0^x z^{a-1}(1-z)^{b-1}\mathrm{d}z$ denotes the lower incomplete Beta function.

Note that an asymptotic T_{W} distribution for real Wishart matrices can be found in [16, Eq. (5.4)], which may be generalized to the complex Wishart case. However, besides being slowly converging, evaluation of this asymptotic result is rather computationally intensive. On the contrary, in the proposed Beta approximation (16) we have established simple *closed-form* relations (14) between the parameters α_0, β_0 and K, N, M in the complex Wishart case.

From the test procedure (5), the resulting approximation to the false alarm probability, for a given threshold ζ, equals

$$P_{\mathrm{fa}}(\zeta) = F_{\mathrm{W}}(\zeta) \approx \frac{B(\zeta;\alpha_0,\beta_0)}{B(\alpha_0,\beta_0)}, \tag{17}$$

where $\zeta \in [0,1]$. Equivalently, for any P_{fa} requirement a threshold can be calculated by inverting $F_{\mathrm{W}}(\zeta)$,

$$\zeta = F_{\mathrm{W}}^{-1}(P_{\mathrm{fa}}). \tag{18}$$

We note that the proposed two-moment Beta approximation corresponds to the simplest form of a general Jacobi polynomial approximation [17]. In the general framework, up to any n-th degree of Jacobi polynomials matching the corresponding n moments of T_{W} would be used. According to the Weierstrass approximation theorem [18], the n moment based approximative false alarm probability becomes exact as the number of polynomials n goes to infinity.

V. Numerical Results

In this section we first investigate the accuracy of the derived approximative false alarm probability by simulations. Then we compare the performance of Wilks' detector with several detectors in scenarios with realistic parameters.

A. Accuracy of the Approximative False Alarm Probability

In Figure 1 we plot the approximative (17) and simulated false alarm probability as a function of the threshold. To quantitatively show the approximation accuracy we calculate the numerical values of the approximation error, measured by the Cramér-von Mises goodness-of-fit criterion [19]

$$\mathbb{E}[T_{\mathrm{W}}^{m}] := \int_{\mathbf{R},\mathbf{E}\succ 0} |\mathbf{R}+\mathbf{E}|^{-m} \frac{|\mathbf{\Psi}|^{-N}|\mathbf{\Psi}|^{-M}}{\Gamma_K(N)\Gamma_K(M)} |\mathbf{R}|^{N-K}|\mathbf{E}|^{M+m-K} \mathrm{e}^{-\mathrm{tr}\left(\mathbf{\Psi}^{-1}\mathbf{R}\right)} \mathrm{e}^{-\mathrm{tr}\left(\mathbf{\Psi}^{-1}\mathbf{E}\right)} \mathrm{d}\mathbf{R}\mathrm{d}\mathbf{E} \tag{8}$$

$$= \frac{\Gamma_K(M+m)}{\Gamma_K(M)\,|\mathbf{\Psi}|^{-m}} \int_{\mathbf{R},\mathbf{E}\succ 0} |\mathbf{R}+\mathbf{E}|^{-m} \frac{|\mathbf{\Psi}|^{-N}|\mathbf{\Psi}|^{-M-m}}{\Gamma_K(N)\Gamma_K(M+m)} |\mathbf{R}|^{N-K}|\mathbf{E}|^{M+m-K} \times \mathrm{e}^{-\mathrm{tr}\left(\mathbf{\Psi}^{-1}\mathbf{R}\right)} \mathrm{e}^{-\mathrm{tr}\left(\mathbf{\Psi}^{-1}\mathbf{E}\right)} \mathrm{d}\mathbf{R}\mathrm{d}\mathbf{E} \tag{9}$$

$$= \frac{\Gamma_K(M+m)}{\Gamma_K(M)\,|\mathbf{\Psi}|^{-m}} \mathbb{E}[|\mathbf{R}+\mathbf{E}'|^{-m}], \tag{10}$$

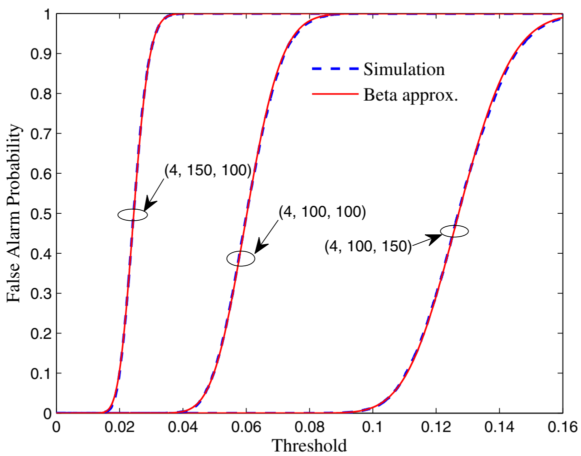

Fig. 1. False alarm probability: analytical (17) versus simulations for different parameter values (K, N, M). From left to right curves, the approximation error is respectively 7.29×10^{-9}, 9.97×10^{-9} and 4.56×10^{-9}.

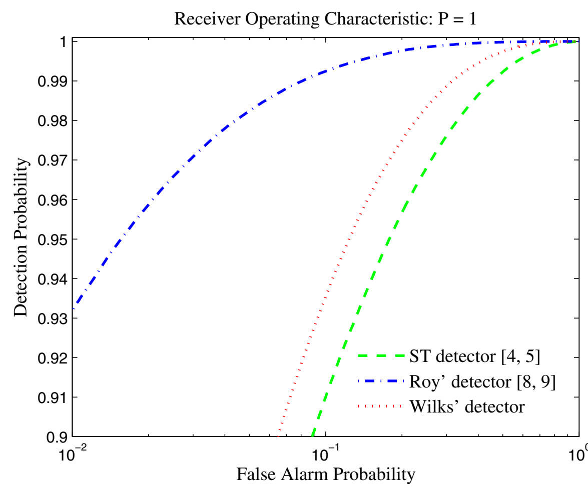

Fig. 2. Performance comparisons: assuming one primary user with $\mathrm{SNR}_1 = -2$ dB using $K = 4$ sensors, $N = 200$ samples and $M = 200$ noise-only observations. The noise correlation is set at $\rho = 0.3$.

$\int_{\zeta}\left|\widetilde{P_{\mathrm{fa}}}(\zeta) - P_{\mathrm{fa}}(\zeta)\right|^2 \, \mathrm{d}\widetilde{P_{\mathrm{fa}}}(\zeta)$, of the proposed false alarm probability (17) with respect to the simulated one $\widetilde{P_{\mathrm{fa}}}$. In Figure 1 we assume uniform sampling in $\zeta \in [0, 0.16]$ with a sampling size 10^6. The results, summarized in the caption of Figure 1, show that the derived analytical false alarm probability (17) matches the simulations well, and the approximation errors are of the same order of magnitude.

B. Performance Comparisons

Here the performance of Wilks' detector is compared with existing detectors by means of Receiver Operating Characteristic (ROC) curves. As the focus of this paper is blind $\mathbf{\Psi}$ detection, we consider for comparison Roy's detector proposed in [8, 9]. In addition, we consider the ST detector derived from the GLR criterion [4, 5], which is a candidate detector in the presence of multiple primary users. Comparisons with other non-blind $\mathbf{\Psi}$ detectors are excluded in this paper, which can be found e.g. in [5]. For a fair comparison, the ST detector also needs to utilize the available noise-only observations. To this end, we replace $\mathbf{R}$ in the ST detector by the 'whitened' sample covariance matrix $\mathbf{E}^{-1}\mathbf{R}$. This modification is motivated by the fact that $\mathbf{E}$ is the maximum likelihood estimate of $\mathbf{\Psi}$, and for a known $\mathbf{\Psi}$ the ST detector becomes a function

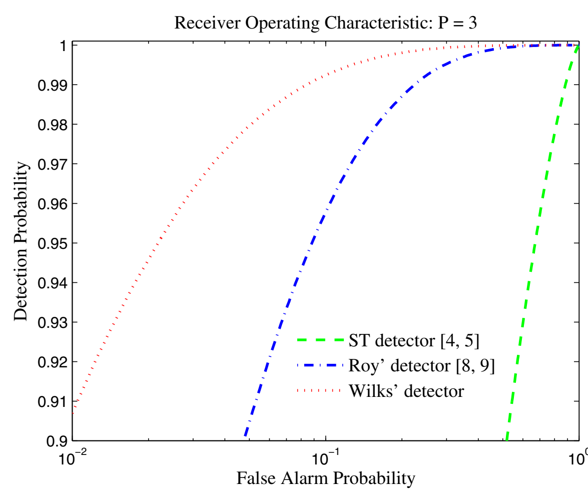

Fig. 3. Performance comparisons: assuming three primary user with $\mathrm{SNR}_1 = -2$ dB, $\mathrm{SNR}_2 = -3$ dB and $\mathrm{SNR}_3 = -4$ dB using $K = 4$ sensors, $N = 150$ samples and $M = 50$ noise-only observations. The noise correlation is set at $\rho = 0.2$.

of $\mathbf{\Psi}^{-1}\mathbf{R}$ [10]. We assume that the entries of the channel matrix $\mathbf{H}$, which are fixed during sensing, are drawn from a standard complex Gaussian distribution. For each ROC curve,

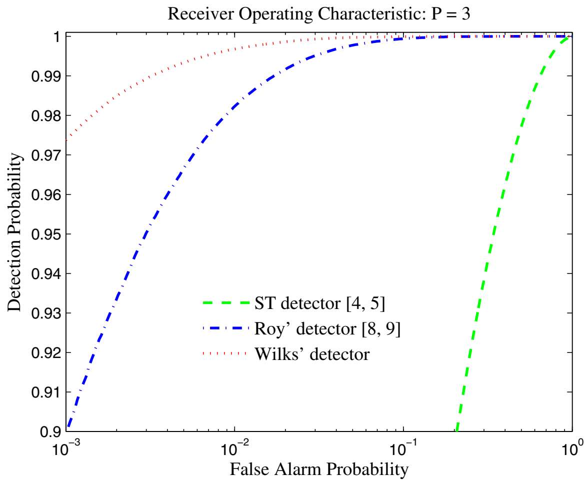

Fig. 4. Performance comparisons: assuming three primary user with $\text{SNR}_1 = -2$ dB, $\text{SNR}_2 = -3$ dB and $\text{SNR}_3 = -4$ dB using $K = 4$ sensors, $N = 150$ samples and $M = 120$ noise-only observations. The noise correlation is set at $\rho = 0.2$.

10^6 realizations of data matrix $\mathbf{X}$ are generated to construct the empirical test statistics distributions under both hypotheses. The exponential correlation model i.e. $(\boldsymbol{\Psi})_{i,j} = \rho^{|i-j|}$, $\rho \in [0, 1)$, is chosen for the noise covariance matrix, where ρ specifies the degree of noise correlation.

We start by studying the simple scenario of a single primary user in Figure 2, where we set $\text{SNR}_1 = -2$ dB, $(K, N, M) = (4, 200, 200)$ and $\rho = 0.3$. For the specific channel realizations considered in Figure 2, the eigenvalues[6] of $\boldsymbol{\Psi}^{-1}\boldsymbol{\Sigma}$ are $[1.68, 1.00, 1.00, 1.00]$. It is seen from this figure that Roy's detector performs best, and indeed its usefulness in detecting single primary user in the case of arbitrary and unknown $\boldsymbol{\Psi}$ has been justified in [8, 9].

We now investigate the case of multiple primary users in Figure 3 and Figure 4, where we consider a scenario of three primary users with $\text{SNR}_1 = -2$ dB, $\text{SNR}_2 = -3$ dB, $\text{SNR}_3 = -4$ dB, and we set $\rho = 0.2$ and $(K, N, M) = (4, 150, 120)$. The same channel realizations are used in both figures, where the eigenvalues of $\boldsymbol{\Psi}^{-1}\boldsymbol{\Sigma}$ are $[1.90, 1.50, 1.17, 1.00]$. In this case we see that Wilks' detector outperforms Roy's detector, which is as expected since the former is designed for multiple P detection when $\boldsymbol{\Psi}$ is arbitrary and unknown. Comparing the two figures we see that an increase of noise-only samples M enlarges the performance gap between Wilks' and the ST detectors, which means that the former is more efficient in using the noise-only samples than the latter does. This is expectable since Wilks' detector was derived from a decision-theoretic criterion i.e. the GLR criterion whereas the modified ST detector here utilizes the noise-only samples in a heuristic manner. Finally we note that our intensive simulations show that both Roy's and Wilks' detectors perform substantially better than the ST detector when $\rho > 0.3$.

[6]The test statistics of Roy's and Wilks' detectors depend on the induced population covariance matrix $\boldsymbol{\Psi}^{-1}\boldsymbol{\Sigma}$ only through its eigenvalues [10].

VI. Conclusion

In this paper, we studied the performance of Wilks' detector, which is a blind-noise-statistics detector in the presence of multiple primary users. A simple and accurate closed-form expression has been derived for the false alarm probability. Simulations show the robustness of Wilks' detector in scenarios with multiple primary users and arbitrary but unknown noise correlation.

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