Cell-edge Inversion by Interference Cancellation for Downlink Cellular Systems

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Abstract—We consider a novel method to improve the downlink data rate of cell-edge users in a cellular system, which is particularly well-suited for Heterogeneous Networks (HetNets). The receivers are assumed to be able to cancel interference by simultaneously processing at most two codewords. Receivers may be served by either the closest cell or a neighboring cell; in the latter case, the receiver cancels the interference from its own cell transmission, and receives the other-cell transmission without this interference. A distributed network utility optimization problem is formulated to exploit this possibility. In a HetNet simulation, where proportional fair network utility is maximized, we observe significant gains for cell-edge users, accompanied by a moderate gain for the network capacity.

I. INTRODUCTION

Interference limits the capacity of modern wireless communication systems. For example, modern cellular communication systems such as 4G LTE are designed to operate with frequency reuse 1. Allowing for hand-over margins, co-channel interference from neighboring cells may require downlink receivers at cell-edge to operate at Signal-to-Interference plus Noise Ratios (SINRs) as small as -7dB [1].

Much current research attempts to mitigate the cell-edge interference problem. Transmission technologies may be improved by attempting multipoint transmission. Multipoint transmission requires sharing accurate channel information between coordinating base stations and user data sequences may need to be transmitted from multiple points in the network. Consequently, the price of implementing multipoint transmission appears to be rather high.

As an alternative, the receivers may be improved, by using Interference Rejection Combining (IRC) receivers, or more advanced Interference Cancellation (IC) receivers. Baseline IRC has been widely studied, and it indeed provides significant gains for cell-edge users in conventional cellular settings, especially when the base stations are deployed with a single transmission antenna [1].

Extending from IRC to full-fledged IC holds much promise. The best coding strategy known for a Gaussian Interference Channel (GIC) is based on Han-Kobayashi rate splitting [2], [3], where IC receivers are combined with cooperative link adaptation by the transmitters. The transmitters split their messages into two parts, one (the public part) intended to be decodable at both receivers, the other (the private part) intended to be decodable only at the intended receiver. The receivers perform Serial Interference Cancellation (SIC), first potentially jointly decoding the public codewords [3], then canceling them before decoding their respective private codeword.

Making practical use of interference cancelation in a large wireless system is challenging, however. A viable method is opportunistic IC, where a receiver cancels interference, whenever it is possible [4]–[6]. Continuing along these lines, it was shown in [7] that in a game-theoretic setting, where Transmitter-Receiver (Tx-Rx) pairs have a strategy space consisting of power control and interference cancellation, it is sometimes beneficial for a selfish user to voluntarily reduce its transmit power so that IC can be exploited.

In a cooperative setting, going past opportunism in IC is beneficial. In [8], a distributed algorithm to provide rate splitting transmissions in a cellular downlink network with SIC-capable receivers was addressed. The ensuing algorithm is complex, as the number of possible orders in which interference from multiple sources can be canceled grows hyper-exponentially in the number of interference sources. To solve the problem of hyper-exponential complexity, it was suggested in [9] to concentrate on single-stage IC, where each receiver can decode at most one interfering signal. A max-min power control problem was addressed, finding the maximum SINR that all receivers in the network of Tx-Rx pairs may enjoy. The problem was shown to be NP-hard. In [10], this approach is generalized to multistage SIC.

In this paper, we address coordinated network IC, i.e. planned IC on the network level, in a downlink cellular system. This is particularly promising in a Heterogeneous Network (HetNet), where there are layers of base stations with different characteristics—small cells have been deployed in the coverage area of a macro cellular network to increase network capacity. In HetNets, macro cells are often overloaded, whereas small cells with limited coverage areas have a small number of users. For this, 3GPP has studied biased cell selection methods, so called cell-range extension [11], and corresponding muting of macro cells, to balance the load between the macro and small cell layers. Network IC provides an alternative way to extend the coverage of small cells, namely by canceling the cell-edge interference from the macro cells.

For network IC, we apply single-stage IC receivers, i.e., the case that a receiver is capable of processing at most two codewords using SIC. This is a logical possibility different from legacy non-interference-canceling receivers, where only the intended codeword may be processed, considering all other transmissions as noise, and from receivers capable of receiving Han-Kobayashi rate-split messages, which would
need to be able to deal with three codewords. Such a possibility may be motivated in part by complexity considerations. We part from [4]–[10] in that we consider a downlink cellular network, and concentrate on the possibility that a receiver may receive transmissions from multiple sources, i.e., base stations. This is the setting of network MIMO and Collaborative Multipoint (CoMP) transmissions [12]. Comparing to CoMP, the difference here is that instead of transmission directivity, we concentrate on the complementary direction of selecting transmission rates while taking the IC capability of the receivers into account. Thus, we consider collaborative multipoint transmission, or soft handover, together with IC-capable receivers in a cellular network.

We assume that all transmitters in the network operate with full transmit power, and that users have been associated with particular cells. A user may receive transmissions from its own cell, as well as from a number of neighboring cells. If receiving a transmission from a neighboring cell, the transmission in its own cell is arranged so that the receiver can cancel the interference from its own-cell transmission. For such a user, the cell-edge has become inverted, so that the user is served by a base station from the other side of the cell edge.

With these assumptions, we formulate a scheduling problem, where resources are allocated to users in the cell so that a network utility is maximized. We show that this problem is convex if the utility function is concave. The problem is however of a high dimensionality, of the order of $N_c(N_u + 1)!$, where $N_c$ is the number of cells, and $N_u$ is the number of users per cell. As a consequence of considering two-stage IC receivers, each user may only receive transmissions from the strongest and second strongest cells, which reduces complexity. The problem allows for distributed formulations, where optimizations for pairs of neighbors are iterated over. However, even with such approaches, the dimensionality of the problem is high, so heuristic approaches are needed even for moderate $N_u$. We simulate the proposed cell-edge inversion technique for load balancing in a HetNet scenario, and show that it is capable of realizing significant gains for cell edge users.

The remainder of the paper is organized as follows. In Section II, the system model is presented, and the concept of cell-edge inversion is introduced. In Section III, the network-level resource optimization problem is discussed, along with a distributed realization. Simulation results in a HetNet scenario are presented in Section IV, and Section V concludes with a discussion.

II. SYSTEM MODEL

A. Cell-edge Inversion

We assume a set of cells $C$, each served by a base station. All base stations transmit with full power. For each cell $c \in C$, the set of neighboring cells is $N_c$, and the set users who would select $c$ as their best cell is $U_c$. With interference cancellation, each user $v \in U_c$ can also receive transmission from a neighbor cell $c' \in N_c$ after canceling the signal from $c$. Thus the network may decide that $v \in U_c$, located at cell-edge in $c$, cancels the signal from its strongest cell $c$, and data to $v$ is transmitted from the second strongest cell $c'$ received by $v$, instead of it conventionally receiving its data from the strongest cell. Here, this concept is called cell-edge inversion. We call $c$ the primary cell of such a user $v$, and the other cell an inversion cell.

Definition 1: A cell-edge inversion is a condition where user $v$ belonging to the coverage area of cell $c$ receives a transmission from neighboring cell $c'$. The transmission in cell $c$ is link adapted so that $v$ may decode this transmission, and cancel it before receiving the transmission from $c'$.

The transmission canceled by $v$ needs not be intended to $v$, but to any user $u \in U_c$. In this case it is said that the transmission to supporter $u$ supports the cell-edge inversion of inverter $v$, which is inverting to cell $c'$. Note that in order to reduce complexity we exclude the possibility that the transmission canceled by $v$ is to a user that has inverted from another cell $c''$ to $c$.

Each transmission can support multiple inverting users, and each inverter could be supported by many potential supporters.

Definition 2: If the supporting transmission in cell $c$ is to the inverting user $v$ itself, the user $v$ is said to be in soft handover (SHO).

Note that the soft handover considered here differs from conventional soft handover in 3G systems, as the receiver does not combine the transmissions from two cells, but decodes them both, using interference cancellation. It is worth noting that with ideal coding and modulation, the data rate supported by such IC-SHO equals the rate achievable with perfect maximum ratio combining of the transmissions from the two cells. Also, it is interesting that the order of decoding does not matter. If $S$ and $S'$ are the received signal powers from cells $c$ and $c'$ at $v$, respectively, and $I_0$ denotes noise and other interference, which is considered Gaussian, we have

$$\log\left(1 + \frac{S}{S + I_0}\right) + \log\left(1 + \frac{S'}{I_0}\right) = \log\left(1 + \frac{S + S'}{I_0}\right).$$

Thus the communication rate achieved with IC-SHO coincides with the rate achieved, e.g., with perfect macro diversity space-time coding. The difference is that in IC-SHO, the transmitting base stations have only to coordinate the data rates used, and the decoupling of the transmissions is left to the receiver.

B. Resources in a Cell

Each cell $c$ has $N = |N_c|$ neighbors. Resources used for transmitting to its own cell users are characterized by the intended receiver $u \in U_c$, and the $N$-dimensional vector $\nu$ of supported transmissions of inversions to the $N$ neighbors. The vector $\nu$ takes values in $\{0\} \cup U_c)^N$, where the entry $\nu_j = 0$ indicates that no user in $c$ uses a transmission in this resource to support an inversion transmission from the $j$th neighbor of $c$. Since each inverting $v$ can receive a transmission from one neighboring cell, each $v$ may be present in $\nu$ at most once.

Possible inversion configurations are thus characterized by the set of vectors $O_c$ where the elements are ordered $N$-element subsets of a set consisting of $U_c$ and $N$ copies of 0. With $U = |U_c|$ being the number of users in $c$, we have $\sum_{n=0}^{\min(N,U)} \binom{N}{n} \binom{U}{n}$ possible configurations in $O_c$. 


For simplicity we assume that all resources in the cell are identical. The information rate per unit resource that is used when transmitting to user $u$ is

$$
\mu_{u,c} = \min_{w \in \{u\} \cup \{e\}} \mu_{u,e},
$$

where $\mu_{u,c}$ is the information rate per unit resource that user $u$ can receive from cell $c$, when no IC is applied. The proportion of resources in cell $c$ that are intended to cell $c'$ own users, supporting inversions $v$, is given by the scheduling weight $w_{uvc}$.

When a user is receiving an inversion transmission from a cell $c'$, the information rate per unit resource is denoted by $\mu_{i,c'}$. The proportion of resources given in cell $c'$ to an inverting user $u$ from another cell is denoted by $w_{uc}$.

The total rate of user $u$ with primary serving cell $c$ is thus

$$
r_u = \sum_{v \in O_c} w_{uvc}\mu_{u,c} + \sum_{c' \neq c} w_{uc}'\mu_{uc}'.
$$

As we consider two-stage IC receivers, we shall only consider inversion configurations in which a user inverts to its second best cell. Only with multistage SIC, would it make sense to receive a transmission from a base station which is not one of the two best.

**III. Resource Optimization Problem**

The objective of the network is to maximize system utility, which is the sum of the user utilities. The user utility is characterized by the function $f = f(r_u)$, which we assume convex and monotonically growing in the user rate $r$. For concreteness, we consider conventional proportional fair utility [13]:

$$
f(r) = \log(r).
$$

**A. Constraints**

The optimization is over the scheduling decisions in the cells, characterized by the scheduling weights $w_{uvc}$ and $w_{uc}$. The scheduling decisions are restricted by

- **Resource constraints:** For each cell $c$, all resources are allocated at most once:

$$
\sum_{u \in U_c} \sum_{v \in O_c} w_{uvc} + \sum_{u \in U_{c'}, c' \neq c} w_{uc}' \leq 1.
$$

- **Support constraints:** For each inverting user $v$, the resource allocated by inverting cell $c_k$ should be overlapped with a supporting resource allocated by serving cell $c$, i.e.,

$$
\sum_{u \in U_c} \sum_{v \in O_{c|v,k}} w_{uvc} \geq w_{vck}.
$$

**B. Distributed Algorithm**

We consider infinitely divisible resources, so that we do not have integer variables in the problem. Then the global optimization problem is convex, but with a high number of variables. The support constraints intertwine the decisions in multiple cells. To cope with the complexity, we distribute the algorithm so that each cell $c$ decides on the resource allocation of the users $u \in U_c$ primarily served by itself. Each cell allocates a fraction of its resources to its neighboring cells, for neighbor-cell users that invert into the cell.

We consider a primal decomposition where all constraints hold with equality. It is straightforward to see that with the system model considered, the resource constraints (4) are fulfilled with equality at a network utility maximum. Furthermore, from (1) it follows that any configuration where the support constraints (5) are not fulfilled with equality, can be mapped to a configuration with the same inversion transmissions $w_{vck}$, but with equality support constraints, and the same or better network utility.

Assuming that support constraints (5) are fulfilled with equality, we may remove the variables $w_{vck}$ from the problem. Then in the resource allocation problem of cell $c$, the derivative of the cell utility $f_c = \sum_{u \in U_c} f(r_u)$ with respect to the resource $w_{uvc}$ becomes

$$
\frac{\partial f_c}{\partial w_{uvc}} = f'(r_u)\mu_{uvc} + \sum_{v_k} f'(r_{vk})\mu_{vck}.
$$

where the sum is over the elements in the vector $v$, i.e. users $v_k$ in cell $c$ that invert to neighboring cell $c_k$, supported by the transmission $w_{vck}$ to $u$, and getting rate $\mu_{vck}$ from the cell-edge inverted transmission.

The resource optimization within cell $c$ can be performed based on cell-utility gradients of the type (6). To properly solve the primal resource allocation within each cell, where resources are given to other cells, the price of the resources has to be taken into account. An infinitesimal increase in $w_{vck}$ incurs an infinitesimal utility loss in cell $c_k$ due to the reduction of resources available to serve the users in $c_k$. We write formally

$$
\Pi_{c'} = -\frac{\partial f_{c'}}{\partial w_{c'}}.
$$

where $w_{c'} = \sum_{u \in U_{c'}} \sum_{v \in O_{c'}} w_{uvc}$ is the sum of all resources in $c'$ used for transmitting to its own users. This is a price for cell $c'$ to give resources to inverting users from other cells.

From these, we can construct the gradient of the network utility with respect to the resources in cell $c$, assuming that all other constraints except the resource constraint (4) in $c$ hold with equality:

$$
\frac{\partial f_N}{\partial w_{uvc}} = \frac{\partial f_c}{\partial w_{uvc}} + \sum_{c_k | v_k \neq 0} \Pi_{c_k}.
$$

Here, $f_N = \sum_c f_c$. This can be used as a distributed gradient or ascent algorithm to maximize the network utility.

It should be kept in mind that the proportionally fair utility function (3) considered here, as well as many other utility functions, is not bounded from below, and the network utility...
is not differentiable at any point where the rate of at least one of the users vanishes.

Accordingly, Lipschitz continuity does not hold, and convergence of gradient-based algorithms can be guaranteed only if there is rigorous control of the absolute step length. The change in the variables should be bounded to be finite. With the utility function (3), the divergences of the gradient happen at a zero-measure part of configuration space, where the network utility is \(-\infty\). This singular subspace is avoided by algorithms aiming at utility maximization.

To simplify the algorithm, we have used a version of normalized steepest descent for the \(l_1\) norm [14]. In the resulting algorithm, the resource with largest \(\frac{\partial f_c}{\partial w_u}\) is incremented, and in each cell involved, the resource with smallest \(\frac{\partial f_c}{\partial w_u}\) is reduced. The algorithm is distributed across the network so that each cell updates its resource allocation periodically, and no neighboring cells update at a given time.

An update in a cell \(c\) involves 1) allocating resources to other cells based on requests, 2) calculating new derivatives \(\frac{\partial f_c}{\partial w_u}\) for all own cell resources, 3) reducing the weight \(w\) of the resource with non-zero \(w\) and smallest derivative, and correspondingly increasing the weight of the resource with largest derivative, 4) signaling the changes in resource requests to other cells 5) calculating a new price \(P_{c, \Pi}\) for the resources given to other cells, and signaling these to neighboring cells.

**Proposition 1:** If the initial point has \(r_u > 0 \forall u\), and an infinitesimal absolute step length is used, the algorithm converges to the unique solution of the convex optimization problem.

**Proof:** The initial point is outside the singular subspace. Each update is based on full knowledge of the consequences of the infinitesimal resource increment on the network utility. Thus each update increases the network utility, the network configuration does not enter the singular subspace, and the algorithm is an ascent algorithm.

With a finite resource increment, one has to add a stopping criterion to the algorithm.

### IV. Simulation Results

To assess the performance of cell-edge inversion, we have performed simulations with a simple heterogeneous network model. We have an hexagonal grid of 12 omnidirectional Macro BS with Inter-Site Distance (ISD) 1 km. In each macro cell, there are three micro BSs, placed close to corners of the cell, at a distance 0.3 ISD from the macro BS. There are 120 UEs dropped uniformly and at random in the network, except that there is a maximum of 10 UEs per cell. Wrap-around boundary conditions are applied. An instance of such a deployment is depicted in Figure 1. The path loss model

\[
L = 37.6 \log_{10} d + 128.1 + 20 \log_{10} f_c
\]

from [15] is used in simulations, where \(d\) is measured in kilometers. Shadow fading and flat Rayleigh fading are applied. Some crucial simulation parameters can be found in Table I.

UEs perform cell selection based on received signal power, so that the best cell \(c\) is the one with the strongest signal. Communication rate is estimated by

\[
\gamma = \frac{P_{\text{MicBS}} G_{\text{MicBS}}}{W N_{\text{Thermal noise level}}} \left(1 + 20 \log_{10} R_{\text{UE}-\text{MicBS}} + 128.1 + 20 \log_{10} f_c\right)
\]

where \(\gamma\) is the SINR of the transmission, and \(\lambda = 2 \text{dB}\) is an implementation loss. A non-optimized distributed algorithm with a step length of 0.0005 has been used to optimize the cell-edge inversion.

Simulation results resulting from resource allocation optimization in 100 instances of the deployment model can be found in Figures 2 and 3, based on information collected from serving 12 000 UEs. The experimental Cumulative Distribution Function (CDF) of the user rate after proportionally fair

**Fig. 1.** Example instance of BS and UE deployment. Triangles: Macro BS. Circles: Micro BS. Dots: UEs.

**Fig. 2.** CDFs of user spectral efficiency (user rate/system bandwidth), proportionally fair scheduling with full cell-edge inversion, with SHO, and with conventional hard handover.
network utility maximization is reported. Results for full cell-edge inversion have the legend “Inversion”, IC-SHO results have the legend “SHO”, and the vanilla system, where no multicell coordination is performed, has the legend “HHO”. We observe that IC-SHO provides nice gains by improving the rate of some 70% of the users, and full inversion provides further gains for these users. In Figure 2 we observe that these gains come with nearly negligible losses for users in good channel states. Figure 3 provides a zoom-in to the experience of cell-edge users. For the users at the 5th percentile of the CDF, often considered as typical cell-edge users, the gains from IC-SHO and cell-edge inversion are significant.

The numerical gains observed in this scenario can be found in Table II. It is interesting to observe that with both IC-SHO and full cell-edge inversion, significant gains in cell-edge performance can be achieved without decreasing the mean throughput, in fact both IC-SHO and full inversion provide a small gain in cell throughput.

V. CONCLUSION

We have explored the potential of using serial interference canceling receivers in downlink cellular systems. User equipment are enabled to cancel transmissions in their strongest cell, and receive transmissions from second strongest cells after the interference from the strongest cell is canceled. As a consequence, in situations where own-cell transmissions suffer from strong interference and the interference is dominantly from one neighboring cell, the user is able to receive a transmission with a high data rate. If the transmission in both cells are to the same user, IC-enhanced soft handover happens, otherwise, we have full-fledged cell-edge inversion.

We studied optimizing a network utility over all cell-edge inversion possibilities, where transmissions from the two best cells are considered for a user. With a convex utility function, the optimization problem is convex, but it is of very high dimensionality. We devised a distributed algorithm based on resource pricing between cells.

The proposed method is well suited for heterogeneous networks, where load asymmetries between large and small cells are the dominant problem in radio resource management. Simulation results in a heterogeneous network show significant gains from both IC-SHO and cell-edge inversion, when proportionally fair network utility is maximized. It is remarkable that the almost 70% gains for the rates of the cell-edge users come with a small gain also in the mean data rate of the users. From this we conclude that considering the possibilities allowed by SIC, and turning part of the interference to useful signal, opens up significant possibilities for improving resource fairness among users in heterogeneous networks.

ACKNOWLEDGMENT

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REFERENCES


**TABLE II. GAINS FROM CELL-EDGE INVERSION.**

<table>
<thead>
<tr>
<th></th>
<th>IC-SHO</th>
<th>Inversion</th>
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</thead>
<tbody>
<tr>
<td>mean user rate</td>
<td>3%</td>
<td>2%</td>
</tr>
<tr>
<td>cell-edge (5%) rate</td>
<td>25%</td>
<td>69%</td>
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Fig. 3. Zoom to the lower end of the user spectral efficiency CDF.